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MATHEMATICAL MODELING OF A RHYTHMIC ELECTRORETINOGRAPHIC SIGNAL FOR STUDYING THE FUNCTIONAL STATE OF THE RETINA IN GLAUCOMA

The article develops a structural-parametric mathematical model of a rhythmic electroretinographic signal that accounts for the characteristics of the retinal electrical response to periodic light stimulation and enables investigation of retinal functional state under normal conditions and in glaucoma. The proposed model is based on an additive representation of the electroretinographic signal, in which each cycle is considered as a superposition of the main physiologically justified components: the early negative a-wave, positive b-wave, high-frequency oscillatory potentials (OPs), and slow C-wave. A system of Gaussian and localized sinusoidal functions is used for the mathematical description of these components, allowing their amplitude-temporal characteristics to be independently specified and controlled.

A distinctive feature of the proposed approach is the introduction of a mathematical framework for accounting for inter-cycle functional adaptation (fatigue) of the retina. The use of an exponential attenuation coefficient enables the nonlinear decrease in component amplitudes during repeated stimulation to be reproduced, which corresponds to actual physiological patterns. In addition, the model provides the possibility of simulating pathological changes characteristic of glaucoma by selectively reducing the amplitudes of those components, namely the b-wave and oscillatory potentials, that reflect the activity of the inner retinal layers affected by this disease.

The practical applicability of the model was confirmed through software simulation in the MATLAB environment for a series of 10 light flashes with the addition of a noise component model. The obtained temporal and spectral results, as well as the plots of signal decomposition and adaptation dynamics, demonstrate the physiological adequacy of the generated signals. The developed mathematical model can be used as a reliable source of verified synthetic data for testing digital signal processing methods, improving automated analysis algorithms, and developing software for modern computer-based systems for the early functional diagnosis of glaucoma.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ РИТМІЧНОГО ЕЛЕКТРОРЕТИНОГРАФІЧНОГО СИГНАЛУ ДЛЯ ДОСЛІДЖЕННЯ ФУНКЦІОНАЛЬНОГО СТАНУ СІТКІВКИ ПРИ ГЛАУКОМІ

У статті розроблено структурно-параметричну математичну модель ритмічного електроретинографічного сигналу, яка враховує особливості формування електричної відповіді сітківки на періодичну світлову стимуляцію та забезпечує можливість дослідження її функціонального стану в нормі та при глаукомі. Запропонована модель ґрунтується на адитивному представленні електроретинографічного сигналу, де кожний цикл розглядається як суперпозиція основних фізіологічно обґрунтованих компонентів: ранньої негативної a-хвилі, позитивної b-хвилі, високочастотних осциляторних потенціалів OPs та повільної C-хвилі. Для математичного опису цих складових використано систему гауссових та локалізованих синусоїдальних функцій, що дає змогу незалежно задавати і контролювати їхні амплітудно-часові характеристики. Особливістю розробленого підходу є введення математичного апарату для врахування процесів міжциклової функціональної адаптації (стомлення) сітківки. Використання експоненціального коефіцієнта згасання забезпечує відтворення нелінійного зниження амплітуди компонентів при багаторазовому повторенні стимулів, що відповідає реальним фізіологічним закономірностям. Крім того, у моделі передбачено можливість імітації патологічних змін, характерних для глаукоми, шляхом цілеспрямованого зменшення амплітуд тих компонентів (b-хвилі та осциляторних потенціалів), які відображають активність уражених при цьому захворюванні внутрішніх шарів сітківки. Практичну придатність моделі підтверджено шляхом програмного моделювання в середовищі MATLAB для серії з 10 світлових спалахів з додаванням моделі шумової складової. Отримані часові та спектральні результати, а також графіки декомпозиції і динаміки адаптації доводять фізіологічну адекватність згенерованих сигналів. Розроблена математична модель може бути використана як надійне джерело верифікованих синтетичних даних для тестування методів цифрової обробки, вдосконалення алгоритмів автоматизованого аналізу та розроблення програмного забезпечення сучасних комп'ютерних систем ранньої функціональної діагностики глаукоми.

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PROBLEM STATEMENT IN GENERAL FORM AND ITS RELATIONSHIP WITH IMPORTANT SCIENTIFIC OR PRACTICAL TASKS

Glaucoma is a progressive disease of the visual system in which disturbances of the functional state of the retina may precede pronounced clinical manifestations and be accompanied by a gradual loss of visual function [1, 2, 17]. This necessitates the development of objective methods for functional retinal assessment and computerized tools for biosignal analysis that enable quantitative evaluation of retinal functional state.

One of the informative methods for functional retinal assessment is electroretinography (ERG), which provides a recording of the retina's summed electrical response to light stimulation. Informative ERG features include the amplitude, temporal, and structural characteristics of individual signal components, which reflect the functional activity of different retinal pathways [3-6, 14, 15]. Under periodic light stimulation, individual ERG responses are sequentially repeated, forming a rhythmic ERG signal. This makes it possible to investigate not only the characteristics of an individual response but also the dynamics of its parameters across a sequence of stimuli, including functional attenuation associated with elevated intraocular pressure [16].

The practical application of rhythmic ERG signals for the computerized assessment of retinal functional state requires the development of an integrated set of digital signal processing methods, algorithms for the automated extraction and evaluation of informative characteristics, and software tools for their implementation. The mathematical model of the ERG signal serves as the key foundation of such an integrated framework, as it defines the signal structure, the set of model parameters, the patterns of their variation, and the capability to generate controlled model data.

The availability of a mathematical model makes it possible to generate controlled rhythmic ERG signals with specified characteristics and use them for the development and validation of digital signal processing methods, assessment of algorithm robustness to noise and parameter variations, and testing of software prior to its application to real biosignals. At the same time, the model should provide the capability to reproduce both the physiologically characteristic structure of an ERG response and controlled changes in its parameters corresponding to the modeled functional state of the retina.

Thus, a relevant scientific and practical task is to develop a mathematical core of an information technology for rhythmic ERG signal analysis, which enables the generation of controlled model signals and provides the basis for the subsequent development of digital signal processing methods, automated analysis algorithms, and software tools for the computerized assessment of retinal functional state in glaucoma.

ANALYSIS OF RESEARCH AND PUBLICATIONS

Mathematical modeling of ERG signals is an important area of research aimed at formalizing the patterns underlying the formation of the electrical response of the retina and reproducing its main structural-temporal and amplitude characteristics. International studies have proposed biophysical and phenomenological approaches to the mathematical description of ERG, relating the parameters of the recorded signal to phototransduction processes and the functioning of individual cellular components of the retina [3-6]. In particular, the works of T. D. Lamb, E. N. Pugh, D. C. Hood, D. G. Birch, L. J. Frishman, A. G. Robson, and other researchers have examined the patterns of ERG response formation and its individual components, providing a theoretical basis for their mathematical formalization [3-6].

A significant proportion of existing approaches comprises models in which the ERG response is represented as a set of structural components with specified amplitude and temporal parameters. Such a representation makes it possible to reproduce the characteristic morphology of the ERG response and investigate the effect of changes in individual parameters on the signal waveform. Within phenomenological modeling, particular importance is attached to the parametric reproduction of the main components of the ERG signal, including the a- and b-waves, as well as oscillatory potentials [3-6]. This principle of mathematical representation is promising not only for signal simulation but also for generating controlled data that can be used to investigate digital signal processing methods and evaluate informative characteristics.

Another research direction is associated with the use of deterministic, stochastic, additive, and periodic models of ERG signals. Ukrainian studies by B. I. Yavorskyi, A. V. Yuzkiv, O. V. Matsiuk, R. A. Tkachuk, M. O. Khvostivskyi, P. O. Tymkiv, V. P. Zabytyvskyi, and their co-authors have considered various approaches to the mathematical representation of electroretinal signals, taking into account their temporal structure, variability, periodicity, and components [7-13]. The use of such models makes it possible to generate model datasets with predefined properties, which is important for investigating biosignal processing algorithms and verifying their performance under conditions of controlled parameter variation.

From the perspective of developing a computerized ERG signal analysis system, the mathematical model serves not only to describe the biosignal but also to form the mathematical core of the subsequent information technology. The structural components and their parameters defined by the model provide the basis for developing digital signal processing methods, identifying informative features, designing automated analysis algorithms, and implementing the corresponding procedures in software. The availability of a controlled model also enables the

generation of test signals with known parameters, making it possible to quantitatively evaluate the accuracy, robustness, and efficiency of the developed methods and algorithms before their application to real ERG recordings.

At the same time, existing models differ both in their principles of mathematical representation and in the level of structural detail. Some approaches are primarily focused on the biophysical explanation of the mechanisms underlying ERG signal formation, whereas others are aimed at reproducing the waveform of an individual electroretinographic response using parametric components. Stochastic and periodic models, in turn, make it possible to account for signal variability and temporal structure [3-13]. However, for the development of a comprehensive software and algorithmic tool for rhythmic ERG signal analysis, it is important to combine these properties within a unified, controllable mathematical representation.

A distinction between an individual ERG response and a rhythmic ERG signal, which is formed by a sequence of individual responses to periodic light stimuli, is of particular importance. A model of an individual response makes it possible to describe its structure and parameters within a single stimulation cycle; however, the development of methods for analyzing a long-duration rhythmic signal additionally requires consideration of the sequence of repeated responses, the time interval between them, and possible changes in their parameters over time.

Periodic repetition of the modeled ERG response alone also does not provide the required functionality. To use the mathematical model as a basis for the development of methods, algorithms, and software, it is necessary to enable controlled variation of the parameters of individual ERG response components and their sequential changes from cycle to cycle. This makes it possible to generate controlled model signals that differ in amplitude-temporal characteristics, the level of inter-cycle variability, and the degree of functional changes in the retina.

Thus, the analysis of existing studies indicates that a theoretical and mathematical foundation for ERG signal modeling has been established; however, the structural-parametric approach to modeling a rhythmic ERG signal as a sequence of individual electroretinographic responses to periodic light stimuli requires further development. A promising approach is to develop a mathematical model that combines the structural description of an individual ERG response, its parametric control, and the generation of a sequence of responses with specified characteristics. Such a model should serve as the mathematical core for the development of digital processing methods for rhythmic ERG signals, algorithms for the automated extraction and evaluation of their informative characteristics, and software for the computerized analysis of retinal functional state in glaucoma.

FORMULATION OF THE OBJECTIVES OF THE ARTICLE

The aim of this article is to develop a mathematical model of a rhythmic electroretinographic (ERG) signal generated in response to a series of periodic light flashes, which accounts for the structural and temporal characteristics of the main components of the ERG response, retinal functional adaptation processes, and controlled changes in signal parameters characteristic of the pathological state associated with glaucoma. The developed model should enable the generation of synthetic ERG signals with specified structural-temporal and amplitude characteristics and serve as a theoretical basis for the subsequent development of digital signal processing methods, automated analysis algorithms, and the corresponding algorithmic and software tools for computerized systems for the functional diagnosis of glaucoma.

PRESENTATION OF THE MAIN MATERIAL

Each light flash initiates an individual electroretinographic response of the retina, which constitutes the basic structural unit of the rhythmic ERG signal. Under periodic light stimulation, a sequence of such responses forms a rhythmic ERG signal. Fig. 1 illustrates the formation of a rhythmic ERG signal as a sequence of ERG responses to a series of periodic light flashes. The response structure includes the a-wave, b-wave, oscillatory potentials, and the slow C-wave.

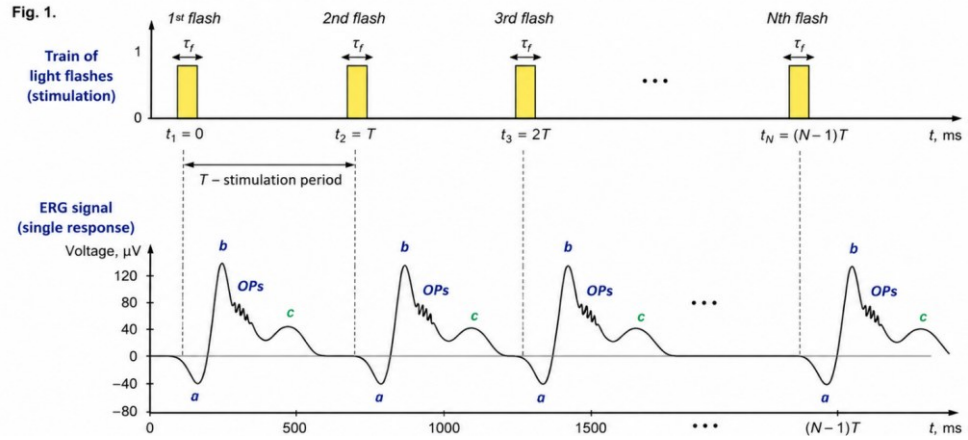

Fig. 1. Formation of a rhythmic ERG signal in response to a series of light flashes: *a* – a-wave (negative phase), *b* – -wave (positive phase), OPs – oscillatory potentials, *c* - c – -wave (slow positive phase), *N* – number of flases in the series, T – interval between flases, τ_f – during of the light flash, t_n=(*N*-1)*T* – time of the last flash in the series

Unlike an individual ERG response, a rhythmic signal reflects not only the functional state of the neural structures of the retina but also their adaptation processes to repeated stimulation, which substantially expands the possibilities for assessing the functional state of the visual system. Therefore, mathematical modeling of a rhythmic ERG signal should account for both the structure of an individual response and the patterns of its changes during repeated exposure to light stimuli.

Structurally, the ERG response to a single light flash contains a sequence of fast and slow components, including the negative a-wave, positive b-wave, oscillatory potentials, the slow C-wave, and recording noise (Fig. 2).

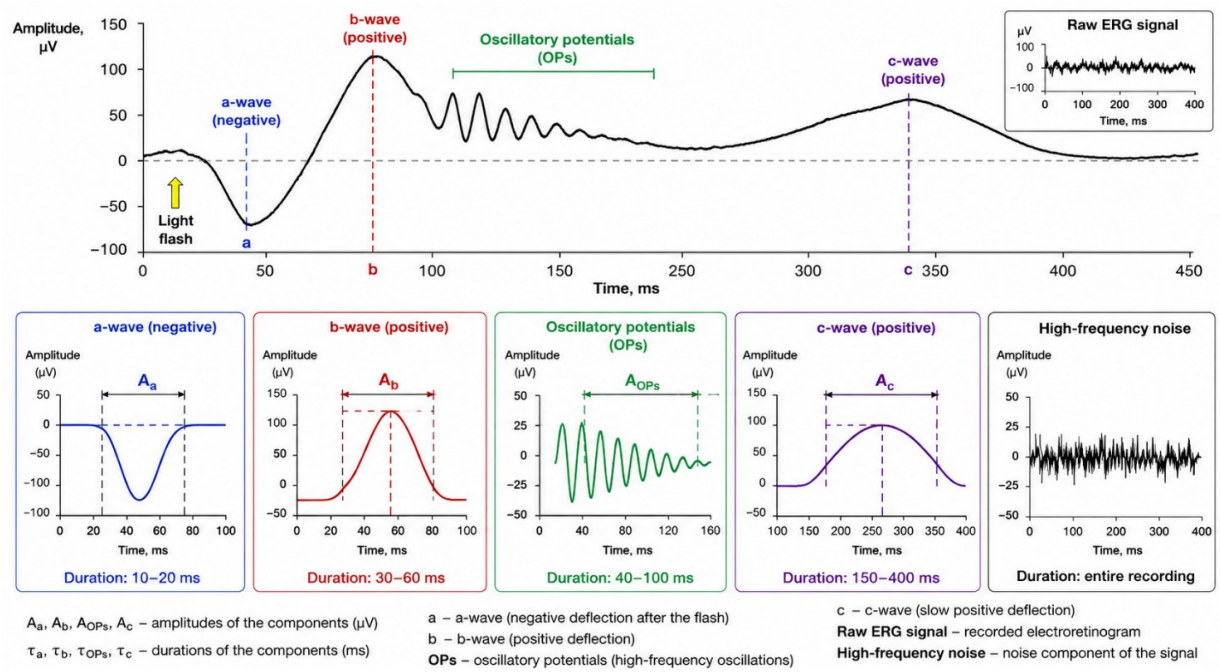

Fig. 2. Structure of a single ERG signal cycle in response to a single light flash and its components

Identification of these components in the mathematical model makes it possible to independently specify their amplitude and temporal characteristics and to investigate the contribution of each component to the formation of the resulting ERG signal.

For the mathematical description of one cycle of the rhythmic ERG signal, an additive representation was used, according to which the resulting response is defined as the sum of the main physiologically relevant components (Fig. 2):

$$R(t) = A(t) + B(t) + O(t) + C(t) + n(t), \tag{1}$$

where $A(t)$ – wave,
$B(t)$ – b-wave,
$O(t)$ – oscillatory potentials,
$C(t)$ – slow C-wave,
$n(t)$ – recording noise,
t – time from the onset of the light flash

This representation makes it possible to consider each component as a separate parameterized signal component and to determine its contribution to the formation of the resulting ERG response. The parameters of the components may vary depending on the specified functional state of the retina, thereby enabling the generation of both a conditionally normal and a pathologically altered signal.

The first component of a single ERG response is the a-wave, which characterizes the early negative phase of the retinal response to light stimulation (Fig. 3).

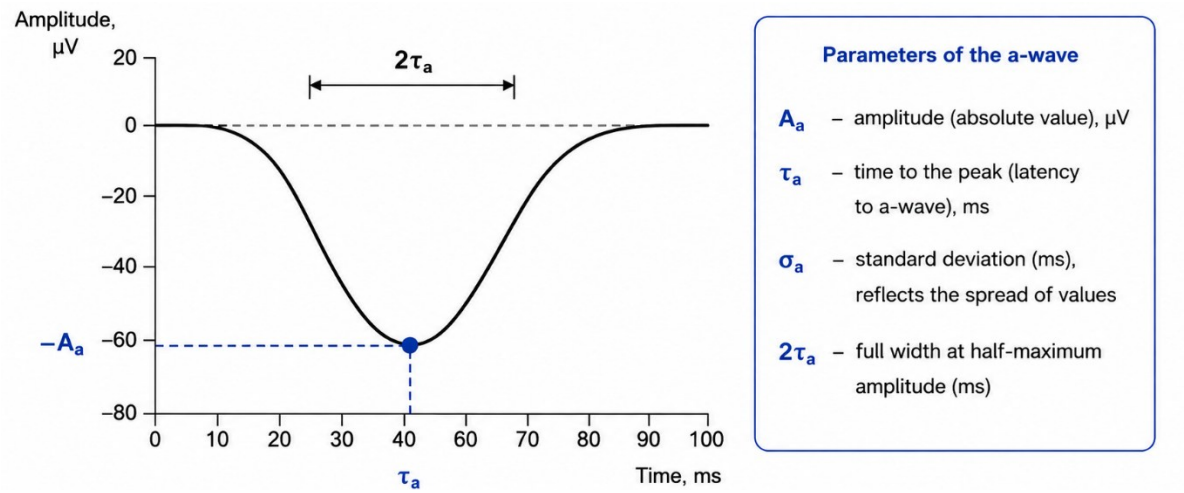

Fig. 3. a-wave – the early negative phase of the ERG response to a light flash

In the mathematical model, it is represented by a negative Gaussian function, whose parameters determine the amplitude, the time to reach the minimum value, and the duration of the corresponding phase:

$$A(t) = A_a \exp\left(-\frac{(t-\tau_a)^2}{2\sigma_a^2}\right), \tag{2}$$

where A_a>0 – a-wave amplitude

τ_a – time to reach its minimum value relative to the onset of the light flash;

σ_a – a parameter determining the temporal width of the component.

The negative sign preceding the amplitude ensures a negative deviation of the signal from the baseline. Varying the parameters A_a, τ_a and σ_a makes it possible to model changes in the magnitude, temporal characteristics, and duration of the early phase of the ERG response, respectively.

The next component of a single ERG response is the b-wave, which is formed after the early negative phase and manifests as a positive deviation of the signal relative to the baseline (Fig. 4).

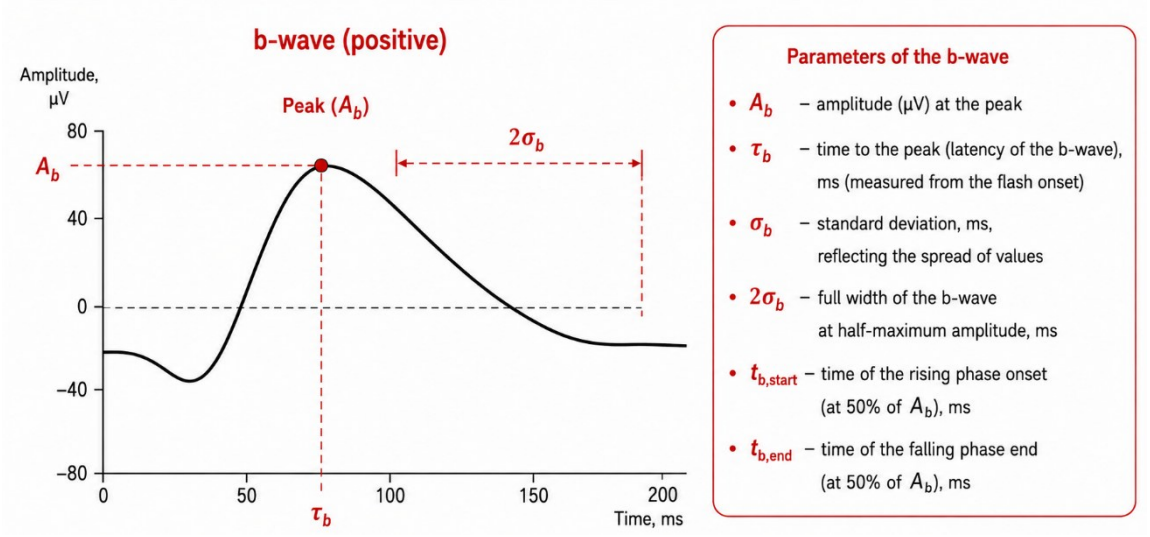

Fig. 4. b-wave – the slow positive ERG response to a light flash

In the mathematical model, the b-wave is represented by a positive Gaussian function:

$$B(t) = A_b \exp\left(-\frac{(t-\tau_b)^2}{2\sigma_b^2}\right), \tag{3}$$

where A_b>0 – b-wave amplitude;

τ_b – time to reach the maximum value of the b-wave relative to the onset of the light flash;

σ_b – a parameter determining its temporal width.

The positive sign of the component produces an upward deviation of the signal following the a-wave. In the model, the parameters A_b, τ and σ_b determine the magnitude of the positive response, its temporal localization, and duration, respectively. Positioning the maximum of the b-wave after the minimum of the a-wave ensures reproduction of the characteristic sequence of the main phases of the ERG response.

Oscillatory potentials (OPs) are additionally distinguished on the rising phase of the b-wave. These are short-duration, high-frequency oscillations that form the microstructure of the ERG response.

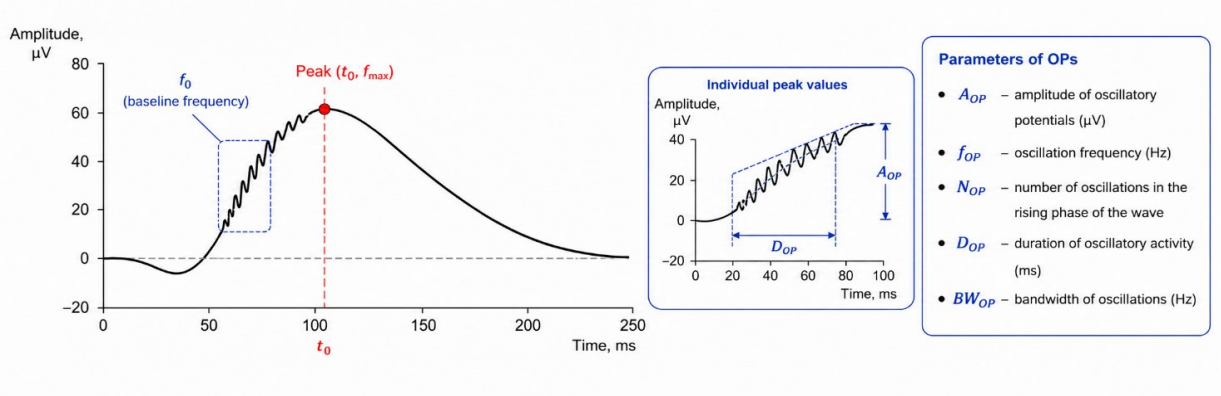

Fig. 5. Oscillatory potentials on the ascending phase of the b-wave

A sinusoidal component localized in the time domain by means of a Gaussian envelope was used for their mathematical representation:

$$O(t) = A_0 \sin(2\pi f_0 (t - \tau_0)) \exp\left(-\frac{(t - \tau_0)^2}{2\sigma_0^2} \right), \tag{4}$$

where A_0 – oscillatory potential amplitude;

f_0 – characteristic frequency of the oscillatory potentials;

τ_0 – temporal position of the center of the oscillation group relative to the light flash;

σ_0 – parameter defining the temporal localization of the oscillatory component.

The use of a Gaussian envelope provides localization of the oscillatory potentials within the rising phase of the b-wave and prevents their spread over the entire duration of the ERG signal. Thus, the OPs form a localized high-frequency microstructure without altering the overall temporal structure of the main a-b response.

The slow component of the ERG response is the c-wave, which characterizes slow changes in the electrical potential of the retina and has a significantly longer temporal duration compared with the main fast components of the ERG response (Fig. 5).

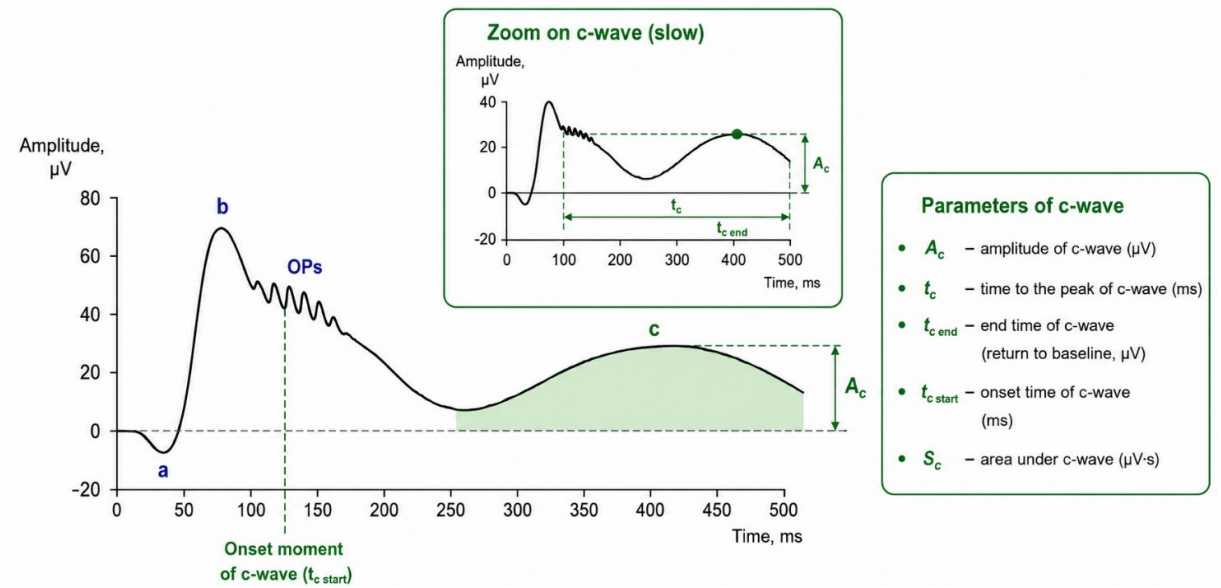

Fig. 5. c-wave – the slow component of the ERG response to a light flash

In the proposed model, the c-wave is represented by a broad positive Gaussian function:

$$C(t) = A_c \exp\left(-\frac{(t-\tau_c)^2}{2\sigma_c^2}\right), \tag{5}$$

where $A_C > 0$ – c-wave amplitude;
τ_C – temporal position of its maximum relative to the onset of the light flash;
σ_C – parameter determining the temporal width of the slow component.

Unlike the fast a- and b-waves and the localized oscillatory potentials, the c-wave is characterized by a significantly longer temporal duration. Therefore, its inclusion in the model makes it possible to account for the slow component of the electroretinographic response and to assess its contribution to the formation of the overall signal during repeated light stimulation.

To generate a periodic ERG signal, individual response cycles are synchronized with the onset of the light flashes. If T is the stimulus repetition period and $t_i=(i-1)T$ is the onset time of the i-th flash, the resulting signal for a series of N stimuli is defined as the superposition of the individual responses:

$$R_{per}(t) = \sum_{i=1}^{N} R_i(t-t_i), \tag{6}$$

where $R_i(t-t_i)$ – the ERG response to the i-th light flash, shifted relative to the beginning of the overall time axis by t_i.

The repetition period of the light flashes is related to their repetition frequency by the relation $T=1/f_s$ where f_s is the light-flash repetition frequency. Under periodic light stimulation, individual ERG responses are sequentially repeated at intervals of T, forming a rhythmic ERG signal whose temporal structure is determined by the number of stimuli and their repetition period.

Given the significantly longer duration of the *c*-wave compared with the a- and b-waves, at a given stimulation frequency, the temporal tails of individual responses may overlap with subsequent cycles. This is taken into account when forming the rhythmic signal as a superposition of individual responses.

Taking into account the structure of the single response, expression (5) can be written in expanded form:

$$R_{per}(t) = \sum_{i=1}^{N}\left[A(t-t_i) + B(t-t_i) + O(t-t_i) + C(t-t_i)\right] + n(t), \tag{7}$$

where $A(t-t_i)$ – a-wave, the negative fast component;
$B(t-t_i)$ – b-wave, the positive component;
$O(t-t_i)$ – oscillatory potentials (OPs);
$C(t-t_i)$ – c-wave, the slow component of the ERG response;
t_i – onset time of the i-th light flash;
N – number of light flashes;
$n(t)$ – additive noise component of the model.

The resulting representation enables the formation of a rhythmic ERG signal as a sequence of similar responses to periodic light stimuli. At the same time, varying the parameters of individual components from cycle to cycle makes it possible to model the functional adaptation of the retina and the differences between conditionally normal and pathological states.

During repeated light stimulation, the parameters of the ERG response may change from cycle to cycle due to functional adaptation of the retina. To account for this effect, the model incorporates a gradual decrease in the amplitudes of the main signal components with increasing stimulus number. The adaptation coefficient for the *i*-th cycle is defined by an exponential relationship:

$$K_i = \exp\left[-\lambda(i-1)\right], \tag{8}$$

where λ – functional adaptation rate coefficient;
i – light stimulus number.

Taking into account the adaptation coefficient K_i, the amplitude parameters of the a- and b-waves, as well as the oscillatory potentials, for each subsequent cycle are defined as follows:

$$A_{a,i} = A_{a,0} K_i , \quad (9)$$

$$A_{b,i} = A_{b,0} K_i , \quad (10)$$

$$A_{o,i} = A_{o,0} K_i , \quad (11)$$

$$A_{c,i} = A_{c,0} K_i , \quad (12)$$

where $A_{a,0}$, $A_{b,0}$. $A_{o,0}$ and $A_{c,0}$ – initial amplitudes of the corresponding components in the first cycle.

This approach provides a gradual decrease in the amplitude of the modeled ERG response as the number of consecutive stimuli increases and makes it possible to reproduce the effect of functional retinal adaptation within a given series of light flashes.

To investigate pathological changes in the ERG signal associated with glaucoma, the mathematical model provides separate parametric representations of the normal and pathological functional states of the retina. The main criterion for modeling the pathological state is a reduction in the amplitude of the ERG response components, which makes it possible to generate a signal with characteristics that differ from those of the conditionally normal response.

For the *i*-th cycle, the amplitude of the b-wave under the pathological condition is determined by the following relationship:

$$A_{b,i}^{path} = k_b A_{b,0} K_i , \quad 0 < k_b < 1 , \quad (13)$$

where k_b – b-wave amplitude reduction coefficient under pathological conditions.

The change in the oscillatory component is considered separately:

$$A_{O,i}^{path} = k_b A_{O,0} K_i , \quad 0 < k_O < 1 , \quad (14)$$

where k_o – coefficient of change in the amplitude of the oscillatory potentials under the pathological condition.

Thus, the pathological ERG signal is formed using the same structural model as the conditionally normal signal, but with modified parameters of individual components. This makes it possible to preserve a unified mathematical structure of the model and investigate the effects of pathological changes by controlled variation of its parameters.

Under real-world ERG recording conditions, the signal is accompanied by bioelectric and measurement noise caused by the activity of other biological sources, muscle electrical activity, patient movements, and the characteristics of the recording system. To make the mathematical model more representative of practical recording conditions, a random noise component is introduced into the generated useful signal.

The recorded signal is defined as follows:

$$R_{reg}(t) = R_{per}(t) + n(t) , \quad (15)$$

where $R_{per}(t)$ – modeled periodic ERG signal;

n(t) – random noise component.

To model background noise, a normally distributed random process with zero mean is used:

$$n(t) = N(0, \sigma_n^2) , \quad (16)$$

where σ_η – root-mean-square value of the noise component.

The introduction of noise makes it possible to assess the robustness of the proposed model and subsequent processing algorithms to the influence of random disturbances. The noise parameters are specified independently of the parameters of the physiological ERG components, enabling separate investigation of the effect of the signal-to-noise ratio on the modeling results.

For practical implementation of the proposed mathematical model as software, a flowchart of the rhythmic electroretinographic signal modeling algorithm was developed (Fig. 7). The algorithm involves sequential generation of individual ERG responses to periodic light flashes, their parametric modification taking into account functional adaptation and pathological changes, superposition of the responses, and formation of the recorded signal with consideration of the noise component.

According to the developed algorithm, the parameters of the light stimulation and ERG response are first specified, after which the components of the a-wave, b-wave, oscillatory potentials, and c-wave are generated for each cycle. The generated responses are sequentially shifted according to the stimulation onset times and combined to form

a rhythmic ERG signal. At the final stage, a noise component is added to the modeled signal, resulting in a signal that approximates real-world recording conditions.

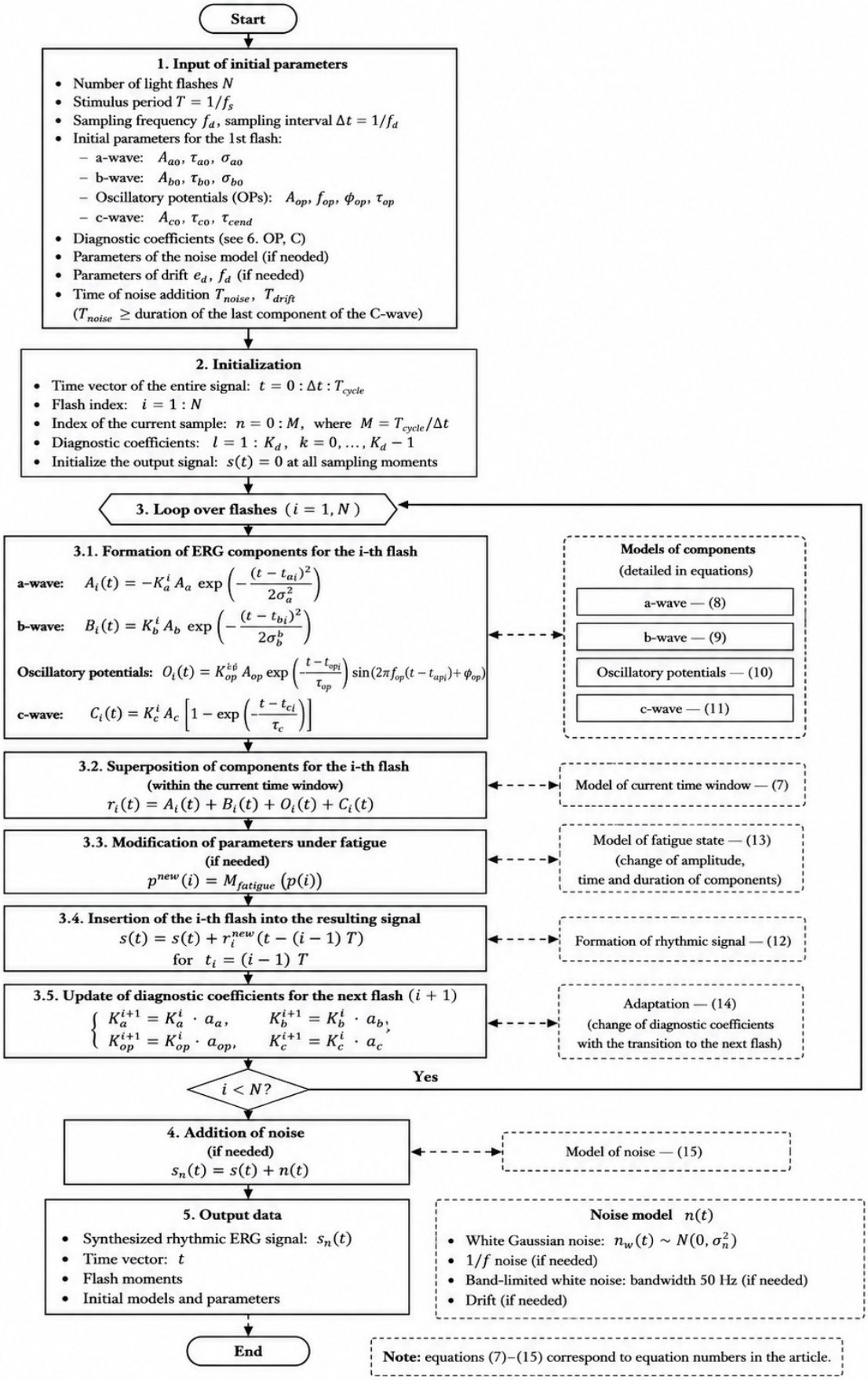

Fig. 7. Flowchart of the algorithm for implementing the mathematical model of the rhythmic ERG signal

Simulation Results of the Rhythmic ERG Signal under Normal and Glaucomatous Conditions

For practical validation of the developed mathematical model, a rhythmic ERG signal was simulated using MATLAB. The simulation was performed for a series of 10 consecutive light flashes at a stimulation frequency of 2 Hz, corresponding to an interstimulus period of 500 ms. To bring the synthesized data as close as possible to real-world biopotential recording conditions, an additive random noise component (white Gaussian noise) was added to the generated signal.

Fig. 8 presents the time-domain waveforms of the modeled rhythmic ERG signal for a conditionally normal functional state of the retina (upper plot) and a pathological state characteristic of glaucoma (lower plot).

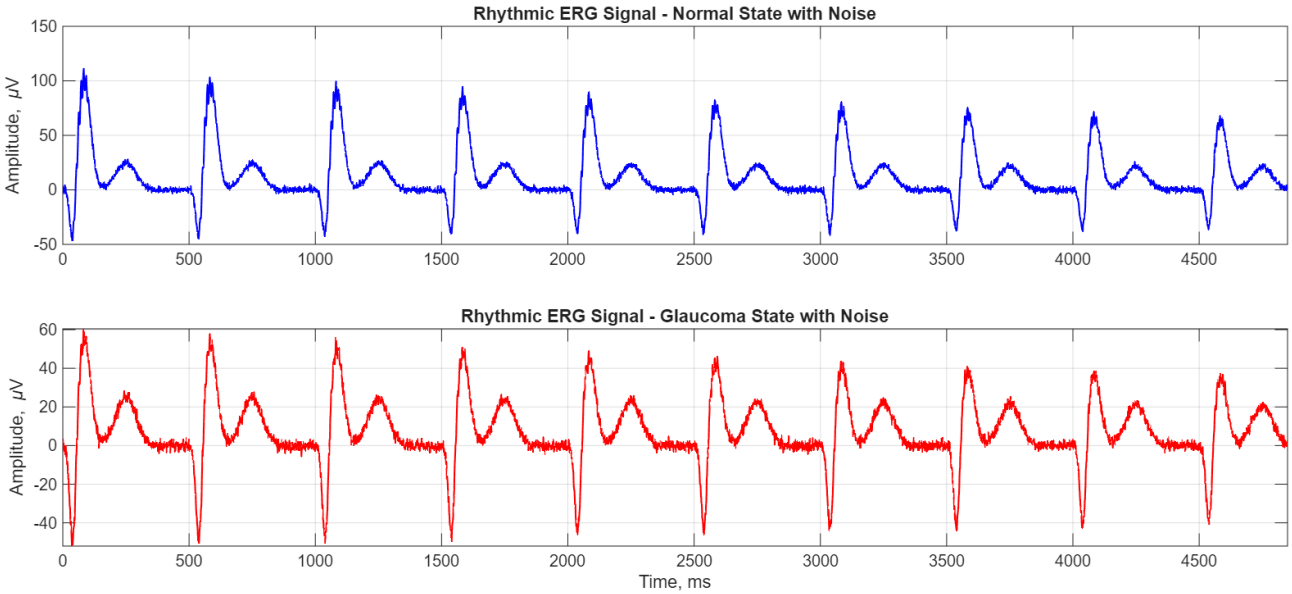

Fig. 8. Time-domain waveforms of the modeled rhythmic ERG signal in response to a series of 10 light flashes

The developed model adequately reproduces the periodic structure of responses to light stimuli. Visual analysis shows an overall decrease in the amplitude of the resulting signal when modeling the pathological condition, which is a direct consequence of the targeted modification of the model parameters corresponding to the activity of retinal layers affected by glaucoma.

A detailed analysis of the structure of a single ERG response (Fig. 9) confirms the correctness of using the mathematical apparatus of signal superposition. The resulting signal (solid black line) is formed as the sum of the fast components (a-wave, b-wave, and oscillatory potentials) and the slow positive component (c-wave).

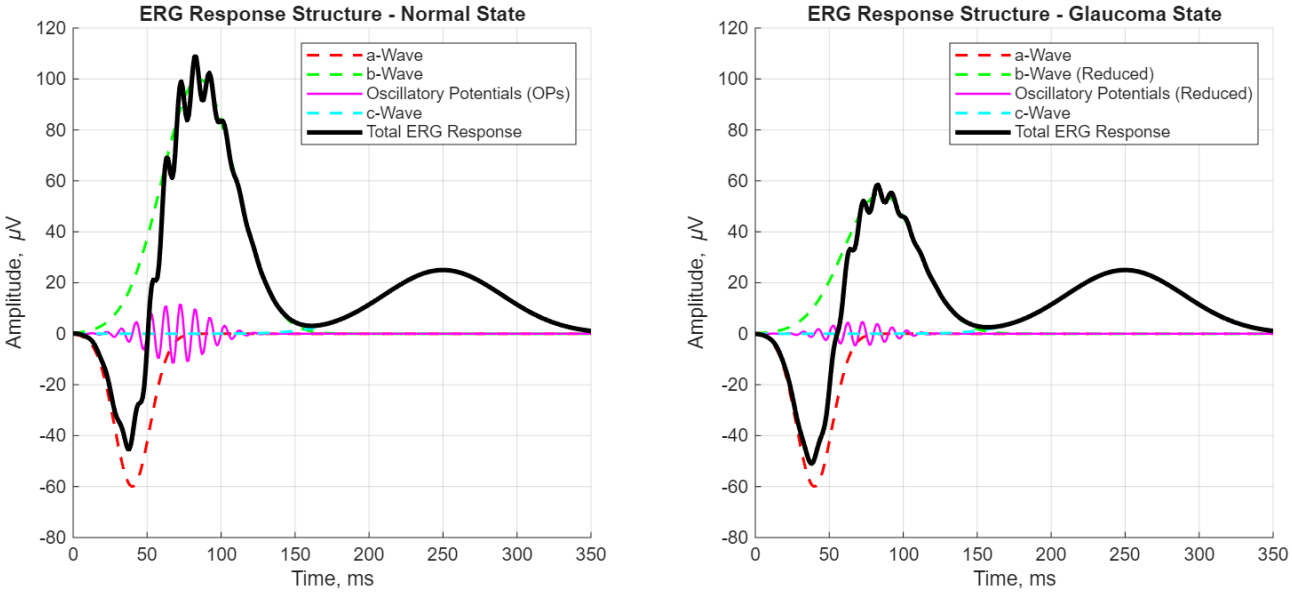

Fig. 9. Additive decomposition and structure of the ERG response within a single stimulation cycle (350 ms)

In accordance with the pathophysiological features of glaucoma development, in the pathological model (right plot), the amplitude of the early negative a-wave (red dashed line) and the slow c-wave (blue dashed line)

remains at the normal level. In contrast, the amplitudes of the positive b-wave (green dashed line) and the high-frequency oscillatory potentials (solid purple line), which reflect the electrical activity of the inner retinal layers, are substantially reduced by the pathological-state coefficients incorporated into the algorithm.

An important advantage of the developed model is its ability to reproduce the processes of functional retinal adaptation (fatigue) to repeated light stimulation. Fig. 10 illustrates the inter-cycle dynamics of the decrease in the peak amplitude of the main diagnostic b-wave.

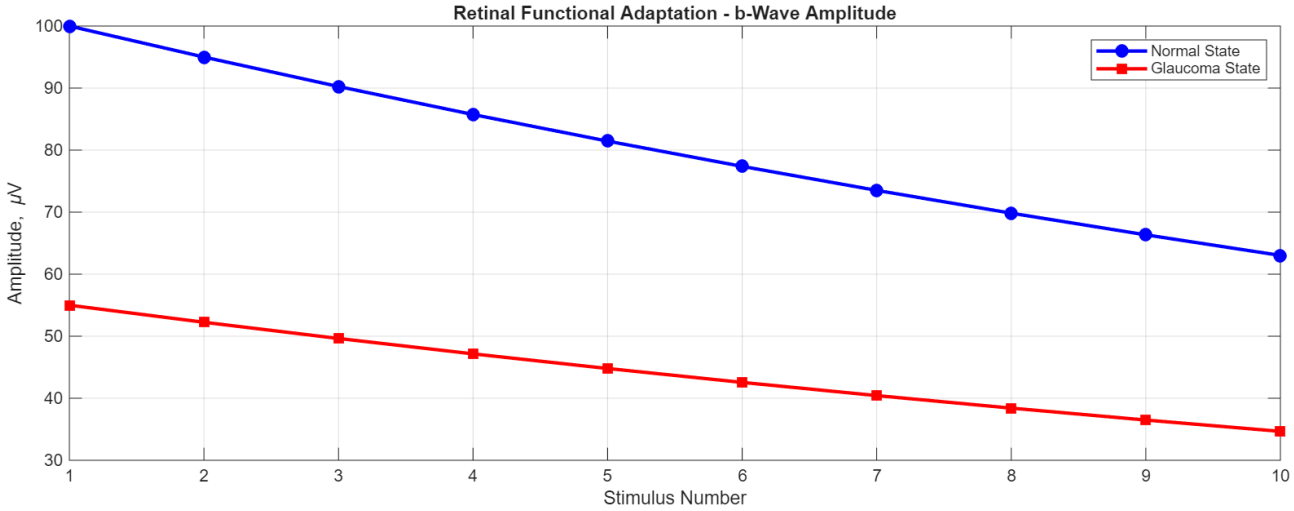

Fig. 10. Dynamics of retinal functional adaptation: decrease in b-wave amplitude from cycle to cycle

By applying an exponential adaptation coefficient, the proposed model demonstrates a controlled gradual attenuation of the response. In the simulated normal condition, the initial amplitude decreases from 100 μV to approximately 63 μV by the tenth stimulus. In the glaucoma model, the initial response is inherently lower (55 μV) and decreases synchronously to 35 μV. This nonlinear decrease in amplitude reflects the patterns of physiological adaptation of neural structures.

The obtained graphical simulation results (Figs. 8-10) fully confirm that the developed structural-parametric mathematical model enables the generation of controlled and physiologically adequate rhythmic ERG signals. The possibility of independently controlling the parameters of individual waves, simulating functional attenuation processes, and introducing pathological changes makes this model a reliable theoretical basis for testing and validating computer-based automated glaucoma diagnosis systems.

CONCLUSIONS OF THE PRESENT STUDY AND PROSPECTS FOR FURTHER RESEARCH IN THIS AREA

As a result of the study, a structural-parametric mathematical model of the rhythmic electroretinographic signal was developed. The model is based on an additive representation of its main physiological components, including the negative a-wave, positive b-wave, high-frequency oscillatory potentials, and slow c-wave. This approach makes it possible to independently control the amplitude and temporal characteristics of each individual component to form a comprehensive resulting ERG response.

In addition, a specific mathematical framework was proposed to account for inter-cycle dynamics and the processes of functional adaptation (fatigue) of the retina. By introducing an exponential attenuation coefficient, the model provides a physiologically adequate representation of the gradual decrease in the amplitudes of the components during serial periodic light stimulation.

An important achievement is the successful implementation of a mechanism for generating a pathologically altered ERG signal characteristic of glaucoma. The use of the introduced pathological-state coefficients enables a controlled reduction in the amplitudes of the diagnostic b-wave and oscillatory potentials, which adequately simulates disturbances in the electrical activity of the inner retinal layers associated with this neurodegenerative disease.

The software implementation of the algorithm and the results of computer simulation for a series of 10 light flashes, performed with consideration of a random noise component, visually and structurally confirmed the adequacy of the developed model. The obtained results demonstrate that the model reliably reproduces both the normal functional state of the retina and the targeted pathological changes.

Regarding prospects for further research, the developed mathematical model provides a reliable theoretical and instrumental basis for generating sets of controlled synthetic ERG signals with predefined functional characteristics. In the future, the model is planned to be used as a reference data source for comprehensive testing, validation, and optimization of advanced digital signal processing methods. It will also serve as a basis for improving

automated analysis algorithms and corresponding software used in modern computer-based systems for the functional diagnosis of glaucoma.

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