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AUTOMATED SYSTEM FOR DETERMINING THE DEFORMATION FIELD OF A LOWER-LIMB RESIDUAL LIMB BASED ON MATRIX PROCESSING OF MEASUREMENT DATA

The paper considers the problem of automated determination of the deformation field of a lower-limb residual limb based on contact measurements and matrix processing of experimental data. The relevance of the study is determined by the need to improve the accuracy of individual prosthesis design, since traditional methods for determining residual limb parameters mainly describe its geometric shape but do not fully account for the deformation response of tissues under local loading. An approach is proposed in which the residual limb surface is represented as a discrete set of contact points corresponding to the cells of a matrix measurement system. For each local loading case, a displacement vector of all contact points is formed, and the set of such vectors is combined into an experimental data matrix. After normalization by the applied pressure, this matrix is interpreted as a mutual influence matrix describing the spatially distributed deformation response of the surface. Based on the obtained matrix representation, a method for reconstructing the residual limb geometry and estimating local deformation characteristics is proposed. The developed approach can be used as a mathematical basis for creating automated residual limb analysis systems and for further design of individual prostheses.

Keywords: automated system, lower-limb residual limb, deformation field, contact measurements, matrix data processing, measurement system, smoothing, surface reconstruction, prosthetics.

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АВТОМАТИЗОВАНА СИСТЕМА ВИЗНАЧЕННЯ ПОЛЯ ДЕФОРМАЦІЙ КУКСИ НИЖНЬОЇ КІНЦІВКИ НА ОСНОВІ МАТРИЧНОГО ОПРАЦЮВАННЯ ВИМІРЮВАЛЬНИХ ДАНИХ

У статті розглянуто проблему автоматизованого визначення поля деформацій кукси нижньої кінцівки на основі контактних вимірювань і матричного опрацювання експериментальних даних. Актуальність дослідження зумовлена необхідністю підвищення точності індивідуального проєктування протезів, оскільки традиційні методи визначення параметрів кукси переважно описують її геометричну форму, проте не повною мірою враховують деформаційний відгук тканин під дією локального навантаження. Запропоновано підхід, за якого поверхню кукси подано як дискретну множину контактних точок, що відповідають коміркам матричної вимірювальної системи. Для кожного випадку локального навантаження формується вектор переміщень усіх контактних точок, а сукупність таких векторів об'єднується в матрицю експериментальних даних. Після нормування за величиною прикладеного тиску ця матриця інтерпретується як матриця взаємного впливу, що описує просторово розподілений деформаційний відгук поверхні. На основі отриманого матричного подання запропоновано метод реконструкції геометрії кукси та оцінювання локальних деформаційних характеристик. Розроблений підхід може бути використаний як математична основа для створення автоматизованих систем аналізу кукси та подальшого проєктування індивідуальних протезів.

Ключові слова: автоматизована система, кукса нижньої кінцівки, поле деформацій, контактні вимірювання, матричне опрацювання даних, вимірювальна система, згладжування, реконструкція поверхні, протезування.

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INTRODUCTION

Modern lower-limb prosthetics requires accurate consideration of the individual geometric and mechanical characteristics of a patient's residual limb. One of the key elements of a prosthesis is the prosthetic socket, which directly contacts the residual limb and ensures load transfer between the human body and the prosthetic structure. A mismatch between the residual limb shape and the socket geometry may lead to nonuniform contact pressure distribution, local tissue overload, discomfort, and reduced functional efficiency of the prosthesis [1].

Methods for measuring interface pressure and sensor systems are used to evaluate the interaction between the residual limb and the prosthetic socket, providing information about local loads in the contact area [2, 3].

Experimental systems for measuring interface pressure during walking also confirm the need to analyze not only individual contact points but also the spatial distribution pattern of loading [4]. Numerical modeling, in particular the finite element method, is used to analyze the stress-strain state in the "residual limb — prosthetic socket" system and to assess the influence of socket design parameters on load distribution [5, 6]. A separate issue is the change in the shape and volume of the residual limb, since such fluctuations affect contact pressure distribution, socket fit, and the stability of the interaction between the residual limb and the prosthesis [7].

A separate research direction in the study of biological tissues is associated with the application of biomedical imaging and structural analysis methods. In particular, [8] presents approaches to assessing bone tissue structure during osseointegration, demonstrating the importance of combining measurement methods with subsequent processing of spatial data. At the same time, such approaches are mainly focused on the analysis of internal tissue structure, whereas for prosthetic socket design it is important to determine the contact deformation response of the residual limb surface under local loading.

To transition from separate local measurements to an integral representation of the deformation state, it is advisable to use matrix processing of measurement data. This approach is consistent with methods of discrete inverse problems and system identification, in which object parameters are determined from its response to controlled input actions [9, 10]. This paper proposes a mathematical model and a data processing method for an automated system for determining the deformation field of a lower-limb residual limb based on sequential local loading of measurement cells and the formation of a mutual influence matrix.

MATERIALS AND METHODS

To determine the deformation field of a lower-limb residual limb, a matrix contact measurement system is used, in which the object surface interacts with a set of discrete measuring elements. Each element corresponds to a separate contact point and provides local loading of the corresponding surface region, followed by registration of its spatial position.

The measurement procedure is based on sequential loading of individual system cells. At the initial stage, the baseline surface configuration is recorded under minimum compression.

Then, controlled pressure is applied to each active cell, after which the spatial positions of all contact points are registered. This scheme makes it possible to obtain not only the local response of the active region but also changes in the positions of other surface points.

The input data include the coordinates of contact points in the baseline state, the coordinates of these points after local loading, the value of the applied pressure, and the discretization parameters of the measurement system. Based on these data, the displacements of contact points are determined, a set of displacement vectors is formed for all measurement steps, and an experimental data matrix is constructed.

The results are processed using a matrix representation of the measurement information. To eliminate the influence of different loading levels, the displacement matrix is normalized by the applied pressure values. The obtained normalized matrix is used as a mutual influence matrix characterizing the deformation response of the residual limb surface to local loading.

Mathematical Model and Data Processing Method

The mathematical model of data processing is based on representing the residual limb surface as a discrete set of contact points corresponding to the cells of the measurement system. Since the lower-limb residual limb can be approximately described as an object close to a cylindrical shape, the coordinate system (θ, z) is used for surface parameterization, where θ is the angular coordinate and z is the coordinate along the residual limb axis. The position of a surface point is defined as:

$$r(\theta, z) = \begin{bmatrix} r(\theta, z)\cos\theta \\ r(\theta, z)\sin\theta \\ z \end{bmatrix},$$

where $r(\theta, z)$ is the radial function that defines the surface shape.

To transition to a discrete model, a grid of contact points is introduced. Let N_θ denote the number of cells along the circumference and N_z the number of levels along the height. Then the total number of measuring elements is defined as:

$$N = N_\theta \cdot N_z$$

Each cell corresponds to a contact point r_{ij}, where $i = 1, \dots, N_\theta$, $j = 1, \dots, N_z$. The baseline position of the contact point under initial compression is denoted as r_{ij}^0, while the position of the same point after applying local loading to the k-th cell is denoted as $r_{ij}^{(k)}$.

The displacement of a contact point is defined as the difference between its current and baseline positions:

$$u_{ij}^{(k)} = r_{ij}^{(k)} - r_{ij}^0$$

To obtain a scalar deformation characteristic, the normal component of displacement is used. If $n_{ij}$is the unit normal vector to the surface at the contact point, then the normal displacement is defined as:

$$d_{ij}^{(k)} = u_{ij}^{(k)} \cdot n_{ij}$$

This makes it possible to transition from a spatial vector description to a discrete field of normal displacements characterizing the deformation response of the surface to local influence. For each measurement step, a displacement vector is formed:

$$d^{(k)} = [d_1^{(k)} \quad d_2^{(k)} \quad \dots \quad d_N^{(k)}]^T$$

After sequential loading of all cells, the obtained vectors are combined into a displacement matrix:

$$D = \begin{bmatrix} d_1^{(1)} & d_1^{(2)} & \dots & d_1^{(N)} \\ d_2^{(1)} & d_2^{(2)} & \dots & d_2^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ d_N^{(1)} & d_N^{(2)} & \dots & d_N^{(N)} \end{bmatrix}$$

The element $D_{ij} = d_i^{(j)}$defines the displacement of the i-th contact point under active loading of the j-th cell.

Thus, each column of the matrix describes the spatial response of the entire system to one local influence, while each row describes the response of a separate contact point to all sequential loadings.

Since the pressure in different cells may have different values, the displacement matrix is normalized by the applied pressure. For this purpose, a diagonal matrix is introduced:

$$P = diag(P_1, \dots, P_N),$$

where $P_k$is the pressure applied to the active cell at the k-th step. The normalized displacement matrix is defined as:

$$\widetilde{D} = DP^{-1}$$

Within the proposed model, this matrix is considered as an estimate of the mutual influence matrix:

$$H \approx \widetilde{D}$$

The matrix $H$is the main object of further processing. Its diagonal elements H_{ii}characterize the local response of the corresponding surface region, whereas the off-diagonal elements H_{ij}, $i \neq j$, reflect spatial interaction between different contact points. This approach makes it possible to consider the deformation field not as a set of independent local measurements but as an interconnected matrix structure.

Based on the obtained data, the surface geometry of the residual limb is estimated. The generalized position of the i-th surface point is defined as the average over all measurement states:

$$r_i^* = \frac{1}{N}\sum_{k=1}^{N} r_i^{(k)}$$

The set of points

$$S^* = \{r_i^* | i = 1, \dots, N\}$$

defines a discrete reconstruction of the residual limb surface. To transition to a continuous description, this set can be approximated by the function $r^*(\theta, z)$.

Local deformation characteristics are determined based on the relationship between displacement and applied pressure. Local compliance can be estimated as:

$$c_i = \frac{d_i^{(i)}}{P_i}$$

The corresponding local stiffness index is written as:

$$k_i^* = \frac{P_i}{d_i^{(i)} + \delta}$$

where $\delta$is a small positive value introduced to avoid division by zero at small displacement values.

The generalized data transformation path in the proposed model can be represented as:

$$\left\{r_{ij}^{(k)}, r_{ij}^0, n_{ij}, P_k\right\} \to u_{ij}^{(k)} \to d_{ij}^{(k)} \to D \to \widetilde{D} = DP^{-1} \to H \to (S^*, K)$$

where $K = \{k_i^* \mid i = 1, \dots, N\}$is the set of local deformation characteristics. Thus, the proposed model provides a complete transition from primary contact measurements to reconstructed surface geometry and estimation of its local deformation properties.

Data Processing Algorithm

The data processing algorithm implements the sequential application of the proposed mathematical model to the results of contact measurements. Its purpose is to transform the primary coordinates of contact points and the values of applied pressure into a mutual influence matrix, reconstructed surface geometry, and local deformation characteristics.

At the initial stage, input data are entered and ordered: coordinates of contact points in the baseline state, coordinates after local loading, surface normals, applied pressure values, and the number of measurement cells. After that, the initial index of the active cell is specified, and an empty displacement matrix is formed, which is then filled with the results of sequential measurement steps.

The main part of the algorithm is implemented as a loop over all cells of the system. For each active cell, the correctness of the applied pressure value is checked. If the pressure is positive, the displacements of contact points relative to the baseline state are calculated, after which their normal components are determined. The obtained values form a displacement vector for the current measurement step, which is written to the corresponding column of the experimental data matrix. If the pressure value is incorrect, the corresponding step is skipped or marked as requiring repeated measurement. The general structure of the data processing algorithm for the matrix contact measurement system is shown in Figure 1.

As a result of the algorithm execution, a set of output data is formed, including the reconstructed surface S^*, the set of local deformation characteristics K, and the mutual influence matrix H. This algorithm ensures the practical implementation of the matrix model and can be used as a basis for further software implementation of an automated residual limb analysis system.

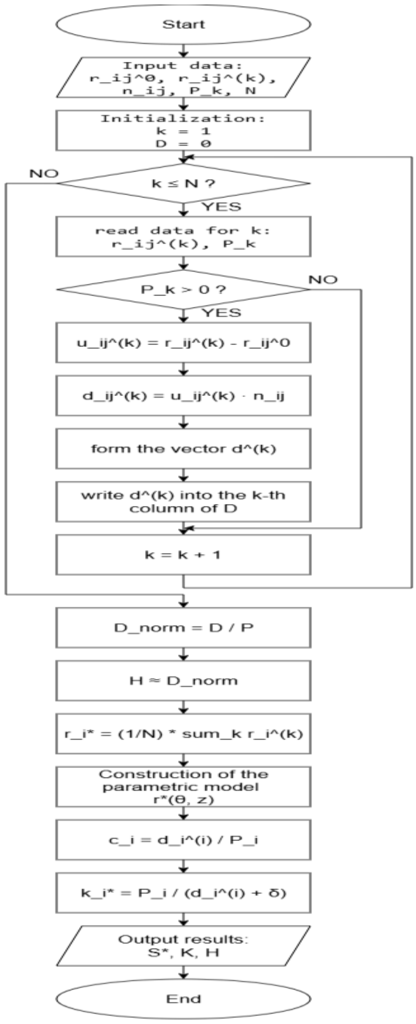

Fig. 1. Block diagram of the data processing algorithm for the matrix contact measurement system

RESULTS AND RELEVANCE

The result of the conducted study is a mathematical model for processing data from a matrix contact measurement system, which ensures the transition from the primary coordinates of contact points and applied pressure values to a generalized description of the deformation response of the residual limb. Unlike approaches in which each point is analyzed independently, the proposed model allows mutual influence between different surface regions to be taken into account.

The main result of processing is the mutual influence matrix H, which is formed based on the normalized displacement matrix. Its diagonal elements characterize the local response of individual contact points to the applied load, while the off-diagonal elements reflect the spatial interaction between different surface regions. Thus, the matrix H can be used as a discrete representation of the residual limb deformation field.

The proposed model makes it possible to obtain the following results:

- reconstructed discrete geometry of the residual limb surface S^*;
- mutual influence matrix H, describing the spatial deformation response;
- local compliance map c_i;
- local stiffness map k_i^*;
- structured representation of measurement data suitable for further software implementation.

The practical value of the proposed approach lies in the fact that contact measurement results can be transformed into a form suitable for automated analysis and further use in the process of prosthetic socket design. In particular, the map of local deformation characteristics can be used to identify regions with increased or decreased compliance, which is important for the rational distribution of load between the residual limb and the prosthetic structure.

A separate advantage is the possibility of combining geometric and deformation descriptions of the surface. While traditional scanning mainly allows the shape of the residual limb to be obtained, the proposed approach supplements geometric information with data on the surface response to local loading. This creates a basis for more accurate individual adjustment of prosthesis parameters.

The proposed model is also convenient for further numerical and software implementation, since all main data are represented as vectors and matrices. This makes it possible to use standard methods of linear algebra, filtering algorithms, regularization, and surface approximation. In the future, the model can be extended by taking into account nonlinear tissue behavior, measurement errors, different loading modes, and individual patient limitations.

The obtained results confirm the feasibility of using a matrix approach for automated determination of the deformation field of a lower-limb residual limb. This approach makes it possible to form an integral representation of the object based on sequential local measurements and can be used as a basis for further development of computer-integrated systems for designing individual prostheses.

CONCLUSIONS

As a result of the conducted study, a mathematical model for processing data from a matrix contact measurement system was developed to determine the deformation field of a lower-limb residual limb. The residual limb surface was represented as a discrete set of contact points corresponding to the cells of the measurement system. This made it possible to formalize the process of sequential local loading and registration of spatial displacements of all contact points.

A matrix representation of measurement data was proposed, in which each column of the displacement matrix describes the response of the entire system to the loading of a separate cell. This approach provides a transition from independent local measurements to a generalized description of the deformation response of the surface. Normalization of the displacement matrix by the applied pressure made it possible to introduce the mutual influence matrix H, which characterizes the relationship between local loading and displacements of different surface regions.

Запропонована модель забезпечує одночасне отримання реконструйованої геометрії поверхні кукси та The proposed model provides simultaneous reconstruction of the residual limb surface geometry and estimation of local deformation characteristics, in particular compliance and stiffness. The developed data processing algorithm defines the sequence of transformation of primary measurement data into output parameters S^*, K, and H, which creates a basis for further software implementation in automated systems for prosthesis analysis and design.

The practical significance of the proposed approach lies in the possibility of supplementing geometric information about the residual limb with data on its deformation behavior under local loading. This can improve the accuracy of individual adaptation of prosthetic socket parameters and ensure a more justified distribution of load between the residual limb and the prosthetic structure.

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