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JUSTIFICATION OF PARAMETRIC OPTIMIZATION WITHIN THE OPERATIONAL PARAMETERS OF BIOTECHNOLOGICAL SYSTEMS

Calculating the values of the objective function and optimizing technical parameters for technical and biotechnological systems involves difficulties in obtaining an exact solution for the objective function. This is primarily due to the fact that taking into account the structural features of the object under study and, for example, the thermal regimes of electromagnetic interaction when formulating boundary value problems complicates the form of these problems. It then becomes impossible to determine whether these boundary value problems will have a unique solution. To ensure the validity of the boundary value problems, it is necessary either to disregard the structural features of the objects in the modeled systems and neglect external factors affecting the systems, or to establish conditions for the validity of the boundary value problems formulated without averaging the features of the modeled systems, where external factors are taken into account. The implementation of computational mathematical models constructed without accounting for the characteristics of the objects under study will reduce the accuracy of the calculated values of the objective function and, as a result, due to the connection between computational and applied optimization mathematical models, the accuracy of the optimization of its parameters.

This article presents a parametric optimization method designed to improve the performance of a biotechnological system for laser embryo cleavage. Using fundamental solutions to heat conduction equations for a homogeneous iron rod with a pulsed heat source and for a single-layer, homogeneous spherical object subjected to thermal action, the authors obtained first approximations to the laser power acting on the embryo. This was made possible by solving problems of synthesizing the parameters of technical means based on the accuracy of the temperature field. The research presented in this article will enable the solution of the problem of finding optimal parameters for laser emitters that ensure laser cleavage of the embryo.

Keywords: parametric optimization, biotechnological system, fundamental solutions, heat conduction equations, power.

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ОБГРУНТУВАННЯ ПАРАМЕТРИЧНОЇ ОПТИМІЗАЦІЇ В МЕЖАХ ФУНКЦІОНУВАННЯ БІОТЕХНОЛОГІЧНИХ СИСТЕМ

Розрахунок значень функції мети і оптимізація технічних параметрів для технічних і біотехнологічних систем пов'язані зі складностями отримання точного значення для функції мети. Це пояснюється насамперед тим, що врахування особливостей будови досліджуваного об'єкта і, наприклад, теплових режимів електромагнітної дії при побудові крайових задач ускладнить вид крайових задач. Тоді вже не можливо буде визначити, чи матимуть ці крайові задачі єдиний розв'язок. Для того, щоб бути впевненими в коректності крайових задач необхідно або не враховувати особливості будови об'єктів з модельованих систем і знехтувати зовнішніми факторами дії на системи, або отримати умови коректності крайових задач, що побудовані без усереднення особливостей модельованих систем, де враховані фактори зовнішньої дії. Реалізація розрахункових математичних моделей, що побудовані без врахування особливостей досліджуваних об'єктів, зменшить точність розрахованих значень функції мети і, як результат цього, через зв'язок розрахункових і прикладних оптимізаційних математичних моделей, точність оптимізації її параметрів.

В статті обґрунтована параметрична оптимізація для підвищення ефективності функціонування біотехнологічної системи лазерного ділення ембріона. Використовуючи фундаментальні розв'язки рівнянь теплопровідності для однорідного залізного стрижня з імпульсним джерелом теплової дії і для одношарового, однорідного сферичного об'єкта, що піддається тепловій дії, авторами отримані перші наближення до потужності лазерної дії на ембріон. Це стало можливим завдяки розв'язанню задач синтезу параметрів технічних засобів за точністю температурного поля. Наведені в статті дослідження дозволять розв'язати задачу пошуку оптимальних параметрів лазерних випромінювачів, що забезпечують лазерне ділення ембріона.

Ключові слова: параметрична оптимізація, біотехнологічна система, фундаментальні розв'язки, рівняння теплопровідності, потужність.

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FORMULATION OF THE PROBLEM

The field involving the use of laser technology to induce cell division in cattle embryos is highly promising and offers the potential to significantly reduce the time required for cell division and improve accuracy. However, the use of existing specialized laser systems, whether for medical or agricultural purposes, is fraught with significant challenges. This is primarily due to the fact that almost all laser systems currently in mass production are specialized and designed for a clearly defined class of tasks. Such systems are expensive, and modifying and adapting them to the specific class of tasks under consideration significantly increases their overall cost. At the same time, the lack of theoretical methods for analyzing the interaction of laser radiation with early-stage elite cattle embryos, methods adequate for the new research objectives, hampers the development of effective laser devices for embryo splitting.

Based on the fundamental solution of the heat conduction equation, first for a homogeneous iron rod with a pulsed heat source, and then for a single-layered, homogeneous, spherical body, this article derives first-order approximations for the power of the laser source. The authors note that the solution to the differential heat conduction equation from the boundary value problem describing the process of electromagnetic exposure to an embryo is a power function with components from Fourier series. The permissible values of the laser exposure power density are obtained from the constraints on the temperature fields at controlled points within the microbiological object (embryo). Thus, the authors have justified parametric optimization for a specific biotechnological system (an embryo under laser exposure) in the case of using laser spot embryo splitting technology. Note that the search for and screening of local extrema in the temperature field is carried out by solving a series of boundary value problems for heat conduction differential equations that describe the state of the embryo under laser irradiation.

ANALYSIS OF THE LATEST RESEARCH

Conditions for the well-posedness of nonlocal boundary value problems for heat conduction differential equations with random disturbances in the right-hand side have been derived [1, 2]. In [3, 4], mathematical models were developed for the optimization of thermomechanical processes in metal processing. The authors of article [5] proposed an analytical method for solving heat transfer problems during a fire in a steel tank. Mathematical models were constructed for thermal imaging control of metal cutting and turning processes [6]. In the article [7], mathematical models of multithreaded processes were developed to synchronize physical, logistical, and engineering operations. Theoretical information regarding the mathematical modeling and optimization of complex systems is presented [8]. Taking into account the numerical experiments conducted in [9] on the operation of critical infrastructure facilities under conditions of a changing security environment in the region, the authors developed mathematical models to support management decision-making in the field of energy supply to critical infrastructure facilities in this region. Using methods of system dynamics, the authors of the article [10] have developed mathematical models for managing banking systems under current conditions.

The purpose of the work is justification of parametric optimization to improve the performance of a biotechnological system for laser embryo cleavage.

PRESENTING MAIN MATERIAL

The objective of synthesizing the parameters of laser-thermal exposure is to determine the acceptable values of the key parameters that can be adjusted when modifying the technical equipment used in this stage of the technological process [11, 12]. These parameters primarily include: the power of the laser exposure; the geometric parameters of the laser spot, sphere, or segment; the speed of the source; and the nature of the change in the pulse exposure over time. Since the primary criterion for the quality of laser embryo cleavage is ensuring the viability of the embryo fragments, the task of synthesizing the parameters of the technical equipment with regard to the accuracy of the temperature regime is of paramount importance. The essence of the problem is as follows: from the set of permissible technical parameters, it is necessary to find those that would ensure the embryo cleavage process while ensuring that the resulting temperature field at points closest to the cleavage site does not exceed predetermined values.

Specifying the permissible values of the temperature field T at specific control points is directly related to ensuring the viability of the segments of the cleaved embryo [13]. We will consider the laser power and the geometric parameters of the source as the main operating parameters. In general, the temperature field T depends on these parameters and can be represented as:

$$T = T\left(x, y, z, t, P\left(x, y, z, t\right), x_1, y_1, z_1\right), \tag{1}$$

where x, y, z – spatial coordinates;

t – time;

P – laser power;

x_1, y_1, z_1 – the most characteristic geometric parameters of the source, such as the width, length, and depth of the laser’s effect on the embryo.

Given the formulation of the problem of synthesizing operating parameters described above, we arrive at the following system of nonlinear inequalities:

$$\begin{cases} T\left(P\left(x, y, z, t\right), x_1, y_1, z_1\right) \le T, \\ P_{\min} \le P\left(x, y, z, t\right) \le P_{\max} \end{cases} \tag{2}$$

when

$$\begin{cases} x^* \le x_1 \le x^{**}, \\ y^* \le y_1 \le y^{**}, \\ z^* \le z_1 \le z^{**}, \end{cases} \tag{3}$$

where T_j – specified permissible values of the temperature field at various points x^j, y^j, z^j monitoring the temperature distribution and, at the same time $x = x^j$, $y = y^j$, $z = z^j$, $j = 1,\ldots$;

$t = t_j$;

*, ** – the asterisks denote, respectively, the minimum and maximum values of the geometric parameters of the source carrier.

Constraints (3) are very simple, and the optimal parameters can be found, for example, using the dichotomy method or the golden section method. As for constraints (2), to analyze them, one must first find an analytical solution to the corresponding boundary value problem. If the boundary value problem is nonlinear or if the distribution of laser power across the source substrate is nonlinear, then the analysis of constraints (2) becomes more complicated. In this context, it is very important to propose a procedure for finding a first approximation of the desired parameters, in this case, the source power. This procedure is particularly important when, in the initial stage of investigating the system of constraints (2)–(3), constraint (3) is defined based on the limits of possible parameter variations. Then, in constraint (2), we switch to strict equality and consider either a single nonlinear equation, if there is a single control point for the temperature field, or a system of nonlinear equations, if there are multiple control points. In both the first and second cases, it is important to find an initial, albeit rough, approximation of the source power.

We can estimate the power of the laser source using an approximate physical model of thermal processes, which allows us to determine a first-order approximation of the power fairly easily. We will base our analysis on the application of the fundamental solution to the heat conduction equation for a homogeneous rod with a pulsed heat source. At that moment, $t = 0$ in the section from $x_0 - \delta$ until $x_0 + \delta$ a sudden release of heat occurs P_0. We obtain the temperature field distribution in the form:

$$T(x,t) = \frac{P_0}{S\rho c} \cdot \frac{1}{2\sqrt{\pi a t}} e^{-\frac{(x-\xi)^2}{4at}}, \tag{4}$$

where P_0 – desired power of the source;

S – cross-sectional area of the rod;

ρ – material density;

c – specific heat capacity of the material;

a – the thermal conductivity of the material;

$$(x_0 - \delta) < \xi < (x_0 + \delta).$$

Suppose that the point x^* there is only one measurement of the temperature field at the specific point in time we are interested in t_0, and the permissible temperature at the monitoring point T^*, then, based on expression (4), constraint (2) takes the form:

$$T^* \ge \frac{P_0}{S\rho c} \cdot \frac{1}{2\sqrt{\pi a t_0}} e^{-\frac{(x^*-\xi)^2}{4at_0}}. \tag{5}$$

From here

$$P_0 \le \frac{T^* S\rho c}{2\sqrt{\pi a t_0} \cdot e^{-\frac{(x^*-\xi)^2}{4at_0}}}. \tag{6}$$

Using the first-order approximation of power from constraint (6) as a starting point, it is easy to proceed to the use of standard software for solving nonlinear constraints and nonlinear equations. Let us consider the determination of the power of a laser source for laser spot embryo splitting technology. An analytical expression for determining the temperature field of a spherical body of radius R with a spherical pulse source of radius r_0, whose power is uniformly distributed across the source medium can be expressed as:

$$T(r,t) = \frac{2}{r\pi\rho c} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi r}{R}\right) \left[\frac{R}{\pi n} \cdot \sin\left(\frac{n\pi r_0}{R}\right) - r_0 \cdot \cos\left(\frac{n\pi r_0}{R}\right)\right] \times \frac{g_0}{A_n} e^{-A_n(t-t_1)} \left(1 - e^{-A_n t_1}\right), \tag{7}$$

where $A_n = \dfrac{a \cdot n^2 \cdot \pi^2}{R^2}$.

To search for valid values g_0, corresponding to the source distribution density and the permissible radius r_0 for the substrate of the laser-irradiated source, we require that the temperature field (7) satisfy the following constraints at no fewer than two points:

$$T\left(r,t,r_0,g_0\right) \le T_1^*\text{, when } r = r_1^*,\ t = t_1^*, \tag{8}$$

$$T\left(r,t,r_0,g_0\right) \le T_2^*\text{, when } r = r_2^*,\ t = t_2^*, \tag{9}$$

where r_1^*, r_2^* – specified coordinates of temperature field control points at specific time instants t_1^* and t_2^*, respectively.

In this case, the ranges of variation for the desired parameters are specified g_0 and r_0:

$$\begin{cases} 0 < g_0 \le g_0^*, \\ 0 < r_0 \le r_0^*. \end{cases} \tag{10}$$

Based on expression (7), constraints (8) and (9) can be expressed as:

$$T_1^* \ge \frac{2}{r_1^* \pi\rho c}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi r_1^*}{R}\right)\left[\frac{R}{\pi n}\cdot\sin\left(\frac{n\pi r_0}{R}\right) - r_0\cdot\cos\left(\frac{n\pi r_0}{R}\right)\right]\times\frac{g_0}{A_n}e^{-A_n\left(t_1^*-t_1\right)}\left(1-e^{-A_n t_1}\right) \tag{11}$$

and

$$T_2^* \ge \frac{2}{r_2^* \pi\rho c}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi r_2^*}{R}\right)\left[\frac{R}{\pi n}\cdot\sin\left(\frac{n\pi r_0}{R}\right) - r_0\cdot\cos\left(\frac{n\pi r_0}{R}\right)\right]\times\frac{g_0}{A_n}e^{-A_n\left(t_2^*-t_1\right)}\left(1-e^{-A_n t_1}\right). \tag{12}$$

Let’s consider constraint (11). We require that the equality holds within it and express the following from it g_0:

$$g_0 = \frac{T_1^*}{2\pi\rho c\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi r_1^*}{R}\right)\left[\frac{R}{\pi n}\cdot\sin\left(\frac{n\pi r_0}{R}\right) - r_0\cdot\cos\left(\frac{n\pi r_0}{R}\right)\right]\times\frac{1}{A_n}e^{-A_n\left(t_1^*-t_1\right)}\left(1-e^{-A_n t_1}\right)} = T^*\left(r_0\right) \tag{13}$$

.

Substituting g_0 into equation (12), we get:

$$T^{**}\left[r_0, T^*\left(r_0\right)\right] \le T_2^{**}. \tag{14}$$

Thus, to determine the rational value r_0 we must solve the nonlinear inequality (14) subject to the constraints (10) on the maximum possible changes r_0. Let’s consider the equality in constraint (14):

$$T^{**}\left[r_0, T^*\left(r_0\right)\right] = T_2^{**}. \tag{15}$$

Using numerical methods to find the roots of the nonlinear equation, we solve it and select only the real root that satisfies the constraints (10) for the desired value r_0. Substituting the value of the unknown parameter obtained from equation (15) into equation (13), which takes the form:

$$g_0 = T_1^*\left(r_0\right) \tag{16}$$

we will obtain the desired density value g_0 source distribution.

This determines the desired operating parameters r_0 and g_0 for the spot-based embryo division technique. Undoubtedly, given the close correspondence between the physical model discussed in this article and the actual object, the obtained values r_0 and g_0 are also close.

CONCLUSIONS

This paper presents initial estimates of the laser source power, for which constraints on the objective function and the technical parameters of the laser emitters have been formulated. The boundary values for the laser exposure temperature were determined by taking into account the thermal conditions of laser heating of the embryo and the viability temperature of the embryonic cells. The authors solved the problem of justifying parametric optimization to improve the efficiency of the biotechnological system for laser embryo cleavage. Calculations of the optimal values of laser exposure power were performed for the technology of embryo cleavage using a laser spot. It should be noted that although the research is focused on optimizing technical parameters specifically for the biotechnological process of laser embryo cleavage, these studies can be applied to optimize other technical and biotechnological systems under the influence of thermal loading sources. However, in the authors’ opinion, without averaging the thermophysical characteristics of the modeled systems and neglecting certain external influencing factors, it will be impossible to justify the correctness of the boundary value problems with heat conduction differential equations. This will undoubtedly lead to additional errors in the calculated values of the temperature field and the technical parameters being optimized. Furthermore, it is important to note that, due to the structural characteristics of the object under study when subjected to electromagnetic influence, it is impossible to expect an analytical solution. To implement the computational mathematical model, approximate methods will need to be applied, which will reduce the accuracy of both the computational mathematical model and, due to their interdependence, the optimization mathematical model. A solution to this situation would be to apply specialized methods, for example, from the theory of differential operators on the space of generalized functions. This will allow us to obtain correctness conditions not only for the computational mathematical model described in this article, but, in the future, for an entire class of partial differential equations. However, in this case, a stage for verifying the correctness of boundary value problems would need to be included between the stages of developing computational mathematical models and their implementation, which would undoubtedly complicate the implementation of boundary value problems.

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