

<https://doi.org/10.31891/2219-9365-2026-85-17>

UDC 621.3.01

KATSYV Samoil

Vinnitsia National Technical University

<https://orcid.org/0000-0003-1375-5229>

e-mail: katsyvsam@ukr.net

KUKHARCHUK Vasyly

Vinnitsia National Technical University

<https://orcid.org/0000-0001-9920-2726>

e-mail: VKuch@ukr.net

KUCHERUK Volodymyr

Uman National University

<https://orcid.org/0000-0002-6422-7779>

e-mail: vladimir.kucheruk@gmail.com

KULAKOV Pavlo

Uman National University

<https://orcid.org/0000-0002-0167-2218>

e-mail: kulakovpi@gmail.com

CONDITIONS FOR REPRODUCING POWER CONSUMPTION IN ELECTRICAL NETWORKS

The article defines the conditions under which continuous current control of electrical power consumption in large, branched high-voltage electrical networks is possible. The fact is that for complete hardware current continuous control of electrical power consumption, it is necessary to have active and reactive power sensors at each node of the electrical network (at each substation) and their connection with dispatching control centers. For countries with large distances between grid nodes and control centers, this requires huge capital expenditures. Therefore, it is much more expedient to place power and voltage sensors in separate nodes and then calculate the remaining nodes using Kirchhoff's laws. Under these conditions, the system of equations using the nodal potential method becomes non-canonical, that is, the known parameters in it are part of the nodal currents and part of the nodal potentials. Such a system does not always have a unique solution, and it depends on the correct placement of power and voltage sensors. But the placement of sensors must be carried out before calculations, therefore the task of determining the conditions for correct placement of sensors without preliminary calculations is relevant, which is considered in the article.

Keywords: power consumption control, nodal potential method, non-canonical system of equations, state reproduction of complex technical systems, fixed network elements, free network elements, network graph.

КАЦІВ Самуїл, КУХАРЧУК Василь

Вінницький національний технічний університет

КУЧЕРУК Володимир, КУЛАКОВ Павло

Уманський національний університет

УМОВИ ВІДТВОРЕННЯ СПОЖИВАННЯ ПОТУЖНОСТІ В ЕЛЕКТРИЧНИХ МЕРЕЖАХ

У статті визначено умови, за яких можливе безперервне струмове керування споживанням електричної енергії у великих, розгалужених високовольтних електричних мережах з урахуванням обмеженості вимірювальної інфраструктури. Показано, що для реалізації повного апаратного струмового безперервного керування необхідно забезпечити наявність датчиків активної та реактивної потужності у кожному вузлі електричної мережі (на кожній підстанції) та організувати їх надійний інформаційний зв'язок із диспетчерськими центрами керування. Для енергосистем із значними відстанями між вузлами мережі та центрами управління така вимога супроводжується надмірними капітальними витратами на встановлення вимірювальних засобів, канали передавання даних і системи синхронізації, що ускладнює практичну реалізацію суцільного вимірювального покриття. Обґрунтовано доцільність часткового розміщення датчиків потужності та напруги лише у вибраних вузлах мережі з подальшим визначенням параметрів в інших вузлах розрахунковим шляхом на основі законів Кірхгофа та методу вузлових потенціалів. За такого підходу система рівнянь стає неканонічною, оскільки відомими величинами виступають як окремі вузлові струми, так і частина вузлових потенціалів. Встановлено, що подібна система не завжди має єдине розв'язання, а її розв'язуваність і однозначність визначаються структурою мережі, топологією з'єднань, параметрами гілок та просторовим розміщенням вимірювальних засобів.

Показано, що коректність відновлення режимних параметрів мережі залежить від виконання умов спостережуваності та структурної визначеності, які забезпечують лінійну незалежність рівнянь і повний ранг відповідної матриці провідностей з урахуванням вимірюваних вузлів. Сформульовано вимоги до мінімально достатнього набору вузлів із датчиками, що гарантує однозначне визначення вузлових потенціалів і потужностей у мережі без необхідності попереднього чисельного моделювання. Запропоновано підхід до вибору точок вимірювання на основі аналізу графової моделі електричної мережі, який дозволяє забезпечити спостережуваність системи за мінімальної кількості датчиків.

Отримані результати можуть бути використані під час проєктування систем моніторингу та керування режимами електричних мереж, оптимізації розміщення вимірювальних пристроїв, а також у задачах побудови цифрових двійників енергосистем. Запропоновані умови правильного розміщення датчиків дають змогу зменшити капітальні витрати на вимірювальну інфраструктуру, підвищити достовірність розрахунку режимних параметрів та забезпечити можливість безперервного струмового керування споживанням електричної енергії в реальному масштабі часу.

It is clear that, provided that there are free and fixed elements in the system S , the system of equations (2) will change its form somewhat due to the fact that in some equations there will be no free term and a new variable will appear, and in some - on the contrary, some variables will disappear.

It is obvious that the condition for the unique solvability of such a system is a nonzero determinant of its coefficient matrix, or (which is identical) if its transformation by the Gauss method leads to a triangular form.

For systems with a large number of elements, the above-mentioned procedures for checking the conditions for unique solvability of a system of linear equations are a rather laborious process, therefore, the development of non-computable methods for determining the solvability of such a system of equations is of great practical importance. One of such methods is considered below.

Determining the conditions for reproducing the state of systems with linear functional relationships without prior calculations

Since the structure of the system (topological relations between its elements) is most conveniently represented in the form of a graph, all further calculations will be carried out in the language of graph theory.

Almost any system can be represented by a graph, in which the elements of the system are vertices, and the functional connections between them are edges. The sequence of edges of a graph that connects any two vertices is called a chain. Two chains intersect if they have at least one common vertex.

Consider a small system S containing six elements (a, b, c, d, e, f) and whose graph can be represented in the form shown in Fig. 1.

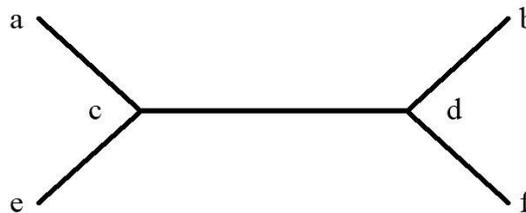


Fig. 1. Graph of system S

Let this system have two fixed and two free vertices (denote them by f and v , respectively). This means that for fixed vertices the parameters x_i, y_i are known, for free vertices both parameters are unknown, for the remaining vertices only the parameter y_i is known.

Let consider two variants for the vertices f and v location on the graph of the system. The vertices of the graph are numbered for the convenience of performing transformations using the Gauss method.

First variant

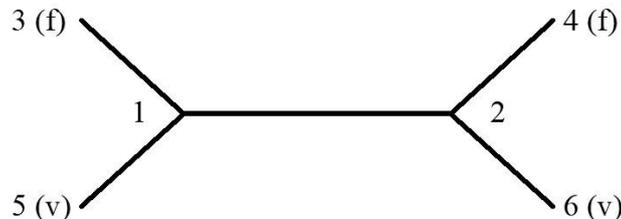


Fig. 2. Variant of the vertices f and v location on the graph of system S , at which the system of equations (2) has the unique solution

The system of equations for this case has the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{15}x_5 &= y_1, \\ a_{21}x_1 + a_{22}x_2 + a_{24}x_4 + a_{26}x_6 &= y_2, \\ a_{31}x_1 + a_{33}x_3 &= y_3, \\ a_{42}x_2 + a_{44}x_4 &= y_4, \\ a_{51}x_1 + a_{55}x_5 &= y_5, \\ a_{62}x_2 + a_{66}x_6 &= y_6. \end{aligned} \quad (5)$$

Since in this variant the parameters $y_1, y_2, y_3, y_4, x_3, x_4$ are known to us, and the parameters $y_5, y_6, x_1, x_2, x_5, x_6$ are unknown, it is advisable to perform the following substitutions:

$$y_1^* = y_1 - a_{13}x_3, y_2^* = y_2 - a_{24}x_4, y_3^* = y_3 - a_{33}x_3, y_4^* = y_4 - a_{44}x_4.$$

After this, system (3) will take the form

$$a_{11}x_1 + a_{12}x_2 + a_{15}x_5 = y_1^*,$$

$$\begin{aligned}
 a_{21}x_1 + a_{22}x_2 + a_{26}x_6 &= y_2^*, \\
 a_{31}x_1 &= y_3^*, \\
 a_{42}x_2 &= y_4^*, \\
 a_{51}x_1 + a_{55}x_5 &= y_5, \\
 a_{62}x_2 + a_{66}x_6 &= y_6.
 \end{aligned} \tag{6}$$

From the third and fourth equations of system (4) it follows:

$$x_1 = \frac{y_3^*}{a_{31}} = \frac{y_3 - a_{33}x_3}{a_{31}}, \quad x_2 = \frac{y_4^*}{a_{42}} = \frac{y_4 - a_{44}x_4}{a_{42}}.$$

Substituting the obtained values x_1, x_2 into the first and second equations of system (6), we obtain:

$$\begin{aligned}
 x_5 &= \frac{y_1^* - a_{11}x_1 - a_{12}x_2}{a_{15}} = \frac{y_1 - a_{13}x_3 - a_{11}\frac{y_3 - a_{33}x_3}{a_{31}} - a_{12}\frac{y_4 - a_{44}x_4}{a_{42}}}{a_{15}}, \\
 x_6 &= \frac{y_2^* - a_{21}x_1 - a_{22}x_2}{a_{26}} = \frac{y_2 - a_{24}x_4 - a_{21}\frac{y_3 - a_{33}x_3}{a_{31}} - a_{22}\frac{y_4 - a_{44}x_4}{a_{42}}}{a_{26}}.
 \end{aligned}$$

Substituting all the obtained values x_1, x_2, x_5, x_6 into the fifth and sixth equations of system (6), we also find the values y_5, y_6 :

$$\begin{aligned}
 y_5 &= a_{51}\frac{y_3 - a_{33}x_3}{a_{31}} + a_{55}\frac{y_1 - a_{13}x_3 - a_{11}\frac{y_3 - a_{33}x_3}{a_{31}} - a_{12}\frac{y_4 - a_{44}x_4}{a_{42}}}{a_{15}}, \\
 y_6 &= a_{62}\frac{y_4 - a_{44}x_4}{a_{42}} + a_{66}\frac{y_2 - a_{24}x_4 - a_{21}\frac{y_3 - a_{33}x_3}{a_{31}} - a_{22}\frac{y_4 - a_{44}x_4}{a_{42}}}{a_{26}}.
 \end{aligned}$$

Thus, for the first variant of the vertices f and v location, the system of equations (5) has a unique solution.

Second variant

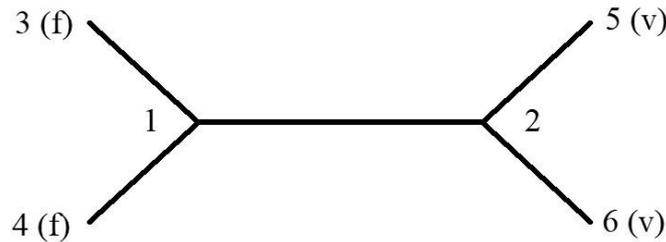


Fig. 3. Variant of the vertices f and v location on the graph of system S, at which the system of equations (2) does not have the unique solution

The system of equations for this case has the following form

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= y_1, \\
 a_{21}x_1 + a_{22}x_2 + a_{25}x_5 + a_{26}x_6 &= y_2, \\
 a_{31}x_1 + a_{33}x_3 &= y_3, \\
 a_{41}x_1 + a_{44}x_4 &= y_4, \\
 a_{52}x_2 + a_{55}x_5 &= y_5, \\
 a_{62}x_2 + a_{66}x_6 &= y_6.
 \end{aligned} \tag{7}$$

Since in this variant the parameters $y_1, y_2, y_3, y_4, x_3, x_4$ are known to us, and the parameters $y_5, y_6, x_1, x_2, x_5, x_6$ are unknown, it is advisable to perform the following substitutions:

$$y_1^* = y_1 - a_{13}x_3 - a_{14}x_4, \quad y_3^* = y_3 - a_{33}x_3, \quad y_4^* = y_4 - a_{44}x_4.$$

After this, system (7) will take the form:

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 &= y_1^*, \\
 a_{21}x_1 + a_{22}x_2 + a_{25}x_5 + a_{26}x_6 &= y_2, \\
 a_{31}x_1 &= y_3^*, \\
 a_{41}x_1 &= y_4^*, \\
 a_{52}x_2 + a_{55}x_5 &= y_5,
 \end{aligned} \tag{8}$$

$$a_{62}x_2 + a_{66}x_6 = y_6.$$

Analyzing the third and fourth equations of system (8), it is easy to see that in the general case (if, of course, $\frac{y_3^*}{a_{31}} \neq \frac{y_4^*}{a_{41}}$) the system of equations is inconsistent. From a topological point of view, the reason for this incompatibility is that the chains connecting pairs of vertices f and v of the graph of the system intersect.

For the general case, we will formulate and prove the corresponding theorem, which will greatly facilitate the solution of applied problems.

THEOREM

Let there be the system S that has fixed and free elements. Let also the number of fixed and free elements coincide and be greater than 1. In this case, if the fixed and free vertices on the graph of this system are arranged in such a way that they cannot be divided into pairs $f - v$, the chains between which do not intersect pairwise, then the system of equations constructed for this graph is inconsistent.

PROOF OF THE THEOREM

Consider a graph of an arbitrary system S , which has 2 fixed and 2 free vertices, which are arranged in such a way that they cannot be divided into pairs $f - v$, the chains between which do not intersect pairwise. Such a graph is not difficult to construct based on the graph shown in Fig. 3, if we add between the vertices (for example, 1 - 3, 1 - 2, 2 - 5, etc.) of this graph a certain number of vertices belonging to the same chains as in the graph (Fig. 3).

First, add vertex q to edge 1 - 2, as shown in Fig. 4.

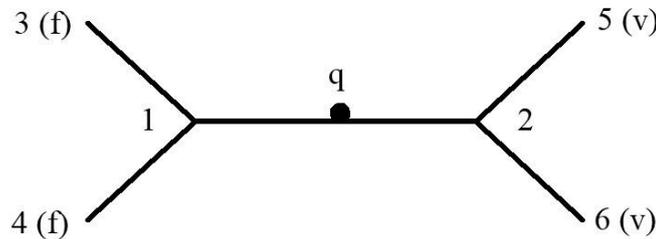


Fig. 4. Variant for an additional vertex in edge 1-2

As a result, one more equation will be added to system (7), and it will take on the following form

$$\begin{aligned} a_{11}x_1 + a_{1q}x_q + a_{13}x_3 + a_{14}x_4 &= y_1, \\ a_{2q}x_q + a_{22}x_2 + a_{25}x_5 + a_{26}x_6 &= y_2, \\ a_{31}x_1 + a_{33}x_3 &= y_3, \\ a_{41}x_1 + a_{44}x_4 &= y_4, \\ a_{52}x_2 + a_{55}x_5 &= y_5, \\ a_{62}x_2 + a_{66}x_6 &= y_6, \\ a_{q1}x_1 + a_{q2}x_2 + a_{qq}x_q &= y_q. \end{aligned} \tag{9}$$

If we make substitutions for this system of equations

$$y_1^* = y_1 - a_{13}x_3 - a_{14}x_4, y_3^* = y_3 - a_{33}x_3, y_4^* = y_4 - a_{44}x_4.$$

then system (9) will take the final form

$$\begin{aligned} a_{11}x_1 + a_{1q}x_q &= y_1^*, \\ a_{2q}x_q + a_{22}x_2 + a_{25}x_5 + a_{26}x_6 &= y_2, \\ a_{31}x_1 &= y_3^*, \\ a_{41}x_1 &= y_4^*, \\ a_{52}x_2 + a_{55}x_5 &= y_5, \\ a_{62}x_2 + a_{66}x_6 &= y_6. \\ a_{q1}x_1 + a_{q2}x_2 + a_{qq}x_q &= y_q. \end{aligned} \tag{10}$$

Obviously, in the case where $\frac{y_3^*}{a_{31}} \neq \frac{y_4^*}{a_{41}}$, the system of equations (8) is inconsistent.

This means that adding vertices to edges that are located after the intersection point of the chains $f - v$ does not make the system of equations consistent if the conditions of the above theorem are met. In other words, from the point of view of unique solvability, the system depicted in Fig. 3 is equivalent to the system depicted in Fig. 4.

Consider the case when the vertex q is added to an edge that is located to the intersection point of the chains f - v , for example, edge 1-3, as shown in Fig. 5.

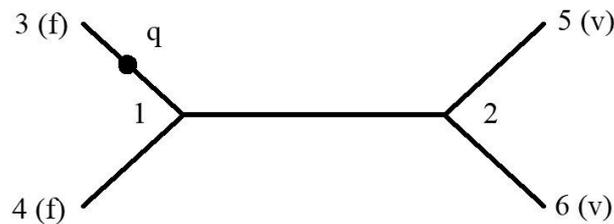


Fig. 5. Variant for an additional vertex in edge 1-3

This system will correspond to the following system of equations:

$$\begin{aligned}
 a_{11}x_1 + a_{1q}x_q + a_{12}x_2 + a_{14}x_4 &= y_1, \\
 a_{21}x_1 + a_{22}x_2 + a_{25}x_5 + a_{26}x_6 &= y_2, \\
 a_{3q}x_q + a_{33}x_3 &= y_3, \\
 a_{41}x_1 + a_{44}x_4 &= y_4, \\
 a_{52}x_2 + a_{55}x_5 &= y_5, \\
 a_{62}x_2 + a_{66}x_6 &= y_6, \\
 a_{q1}x_1 + a_{q3}x_3 + a_{qq}x_q &= y_q.
 \end{aligned} \tag{11}$$

From the third equation it is easy to find x_q

$$x_q = \frac{y_3 - a_{33}x_3}{a_{3q}}.$$

Substituting this value into (9), we obtain

$$a_{q1}x_1 + a_{q3}x_3 + a_{qq} \frac{y_3 - a_{33}x_3}{a_{3q}} = y_q.$$

From this equation it is easy to find x_1 .

$$x_1 = \frac{y_q - a_{q3}x_3 - a_{qq} \frac{y_3 - a_{33}x_3}{a_{3q}}}{a_{q1}}.$$

At the same time, it follows from the fourth equation that $x_1 = \frac{y_4 - a_{44}x_4}{a_{41}}$, which means that in the general case the system of equations (11) is also incompatible, that is, the systems presented in Fig. 3 and Fig. 5 are equivalent from the point of view of unique solvability.

It is obvious that when adding a certain number of vertices to the edges that are located to the intersection point of the chains f - v , it is always possible to perform a certain chain of variable substitutions that will show the incompatibility of the system of equations.

Thus, it can be argued that when the conditions of the theorem are met, any system, from the point of view of unique solvability, is equivalent to the system whose graph is depicted in Fig. 3, that is, its system of equations is inconsistent, which was to be proven.

This theorem makes it possible, without calculating a non-canonical system of equations using the nodal potential method, to determine fixed nodes (vertices) using a topological analysis of the electrical network graph, i.e. nodes that must be equipped with voltage and power measurement devices at the same time. The remaining nodes, which we call free, do not require equipment with any measuring devices.

In this case, the following conditions must be met:

- the number of fixed nodes (f) must be equal to the number of free nodes (v);
- the fixed and free nodes must be arranged in such a way that they can be divided into pairs f - v , the chains between which do not intersect in pairs.

Fixed node selection algorithm

A practical algorithm for selecting fixed nodes without extensive calculations can be constructed using the following steps:

1. Calculating the degrees of nodes. For each node of the network graph, we determine its degree – the number of adjacent edges (lines).

2. Initial selection by degree. Nodes with the highest degree are considered prioritized for "fixed" status because they are incident to a greater number of links and thus "reserve" information about larger portions of the network. So, in the first stage, we assign the status of fixed to all nodes with the maximum degree.

3. Check for intersecting chains. After the initial selection, we check whether fixed nodes form common branches. To avoid crossing "chains" (selected subgraphs), we should next consider pairs of adjacent nodes. We label each pair of neighboring nodes so that each pair has one potential fixed node and one free node. That is, we avoid a situation where two fixed nodes lie on one common chain. This approach ensures that fixed nodes are "scattered" across the network.

4. Choosing among neighboring pairs. For each such pair, we adopt the following rule: we make the node that is closer to the source node (power node) fixed. The smaller graph step from the source to the node provides more reliable data transmission, so the closer node is given the status of "fixed". Thus, of each pair of adjacent nodes, the one that is less distant from the source becomes "fixed," while the other remains "free."

5. Repeat as long as necessary. Continue steps 3 - 4 until the necessary condition is met and it is possible to add new fixed nodes without violating the continuity (connectivity) conditions of the solution. In general, the algorithm ensures that the selected fixed nodes evenly cover the network, and no loops are formed between them.

Let consider an example of applying these conditions to determine fixed nodes in an electrical network, which is shown in Fig. 6. We will assume that the network is powered by node 1.

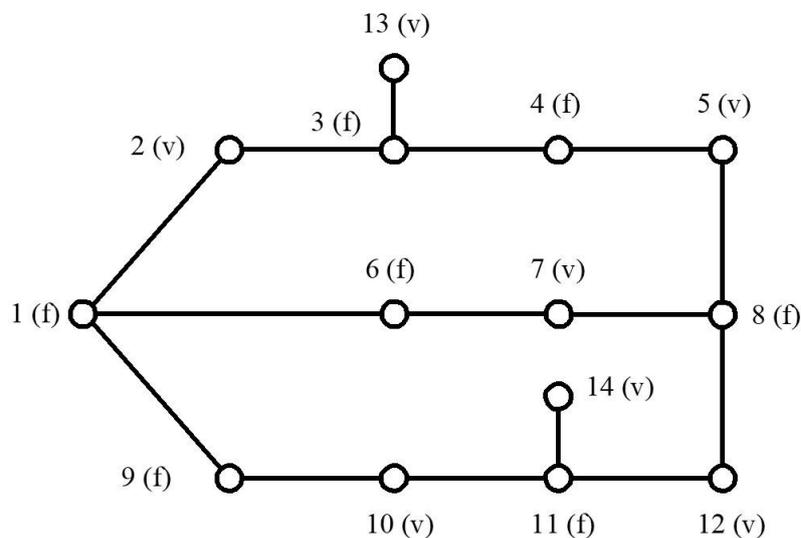


Fig. 6. Fragment of the electrical network

Since the network contains 14 nodes, the number of fixed nodes should be half of this number, i.e. 7. It is advisable to start the procedure of assigning the status f to a node (graph vertex) by determining the degree of the vertex for each of them, that is, the number of graph edges connected to this vertex.

It is necessary to pay attention to the fact that not only devices for measuring node power are installed in the node, but also devices for measuring power on the lines connecting this node with others. Although these parameters are not used directly in the mode calculations, they allow reducing the degree of uncertainty of information in the event of an emergency disappearance of telemetric data from one of the nodes. Therefore, it is necessary to grant the status f primarily to nodes with a higher vertex degree.

In the scheme (Fig. 6), for nodes 1, 3, 8, 11, the degree of the vertex is 3, for the remaining nodes it is 2. Therefore, first of all, we assign the status f to nodes 1, 3, 8, 11.

So it remains to give the status f to three more nodes. To avoid the possibility of crossing chains of f - v pairs, these pairs must consist of adjacent nodes. Therefore, one of the nodes of the pair 4 - 5, 6 - 7, and 9 - 10 can claim the status of f. For further selection, it is advisable to apply other considerations. Node 5 is located at a greater distance from power node 1 than node 4. Then it is advisable to assign status f to node 4, since at a shorter distance the transmission of telemetric information will be more reliable. The same considerations can be applied to node pairs 6 - 7 and 9 - 10. As a result of the analysis, we obtain a table of fixed and free nodes (Table 1).

Table 1

Fixed and free node numbers	
Fixed nodes	Free nodes
1, 3, 8, 11, 4, 6, 9	2, 5, 8, 10, 12, 13, 14

The node statuses are shown in the electrical network scheme (Fig. 6).

This provides significant savings in capital costs for substation equipment, mainly due to savings on communication equipment, and to a lesser extent due to a reduction in the number of measuring instruments.

Conclusions

1. To ensure continuous monitoring of electrical power consumption, it is necessary to have active and reactive power sensors at each node of the electrical network and their connection to dispatching control centers, which for large networks requires unreasonably large capital costs. Therefore, it is proposed to place power and voltage sensors only in individual (fixed) nodes and then reproduce all unknown parameters of the network mode according to Kirchhoff's laws.

2. Under such conditions, the system of equations using the nodal potential method becomes non-canonical, that is, only a part of the nodal currents and a part of the nodal potentials are known parameters, and if the measuring instruments are incorrectly arranged, it may not have the solution at all. Therefore, their placement must be carried out without prior calculations.

3. The theorem developed and proven by the authors allows for the correct placement of measurement tools through topological analysis of the network using graph theory. Its adequacy has been confirmed by the implementation of the hardware and software complex "Analytical System for Reproducing Electricity Consumption" in several divisions of the energy supply enterprise.

References

1. Katsyv, S. Application of non-Euclidean metric in the electric power industry for reduction of measurement uncertainty / S. Katsyv, V. Kukharchuk, V. Madyarov, V. Kucheruk, P. Kulakov, M. Gribov // Bulletin of the Karaganda University. «Physics» series., № 1(113) / 2024, p. 83 – 91; <https://doi.org/10.31489/2024PH1/83-91>
2. Karpov Yu.O., Kukharchuk V.V., Vedmitskiy Yu.H., Katsyv S.Sh. Teoretychni osnovy elektrotekhniki. Ustaleni rezhymy liniinykh elektrychnykh kil iz zoseredzhenymy ta rozpodilenyymy parametramy: Pidruchnyk. – Kherson: OLDI-PLIuS, 2014. – 325 s.
3. Katsyv S.Sh., Mokin B.I. Matematychni modeli determinizatsii protsesiv v systemakh elektropostachannia: Monohrafiia. – Vinnytsia: UNIVERSUM-Vinnytsia, 2005. – 114 s.
4. Gamm A.Z., Golub I.I. Nabludaemost elektroenergeticheskikh sistem. – Moscow: Nauka, 1990. – 200 s.
5. Lezhniuk P.D., Sierova I.A., Nguien Dang Tam. Formuvannia stratehii poetapnoho rozvytku system zboru i peredachi informatsii v enerhosystemakh // Visnyk Vinnytskoho politekhnichnoho instytutu. – 1994. – No. 1. – P. 63–68.
6. Mokin B.I., Kaminskyi V.V., Katsyv S.Sh. Systema vidtvorennia rezhymu elektrospozhyvannia v umovakh nedostatnoi kilkosti telemetrychnoi informatsii // Visnyk Vinnytskoho politekhnichnoho instytutu. – 1999. – No. 2. – P. 63–65.