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IMPROVED METHOD FOR GENERATING STOCHASTIC SIGNALS USING VOLTERRA SERIES

The article presents an improved method for forming and nonlinearly processing stochastic signals in command-and-control information systems, based on second-order Volterra series with adaptive kernel parameter tuning. A mechanism for adjusting the kernel structure in accordance with the statistical properties of the input signal is proposed, enabling increased model stability under varying spectral content.

To evaluate the effectiveness, an experimental comparison was conducted between the improved method and the baseline Volterra-series approach, analyzing indicators such as effective spectral width, spectral-efficiency gain, impact on high-frequency components, and performance on structured (OFDM) and noise-like signals. The optimization reduced the effective spectral width by up to 8.6% for structured signals and ensured a consistent spectral-efficiency improvement of 1.9–5%, while preventing the uncontrolled spectral expansion characteristic of the basic model when processing noise realizations. The experimental results confirm the feasibility of integrating the adaptive kernel-tuning mechanism into the Volterra-based processing pipeline to improve its efficiency for real-time operation under limited frequency-resource conditions.

Keywords: Volterra series; stochastic signals; nonlinear systems; spectral efficiency; memory effects; adaptive signal processing; nonlinear distortion compensation; effective spectral width; energy concentration; wideband signals; spectral redundancy reduction.

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МЕТОД ФОРМУВАННЯ СТОХАСТИЧНИХ СИГНАЛІВ ІЗ ВИКОРИСТАННЯМ РЯДІВ ВОЛЬТЕРА

У статті запропоновано удосконалений метод формування стохастичних сигналів для інформаційно-керуючих систем на основі використання рядів Вольтера другого порядку з адаптивним налаштуванням параметрів ядра відповідно до статистичних характеристик вхідного сигналу. Актуальність дослідження зумовлена необхідністю підвищення спектральної ефективності та стійкості до завад у сучасних радіотехнічних системах в умовах обмеженого частотного ресурсу та наявності нелінійних спотворень і ефектів пам'яті каналу. На відміну від класичних лінійних методів обробки, запропонований підхід забезпечує урахування внутрішніх нелінійних взаємозв'язків між відліками сигналу та дає змогу керовано формувати його спектральну структуру.

Розроблено узагальнений адаптивний алгоритм, що поєднує попередню нормалізацію та центрування сигналу, побудову моделі Вольтера, локалізовану та глобальну реконструкцію, регуляризацію та тензорну факторизацію ядра, а також ітераційне оновлення коефіцієнтів за критерієм мінімуму середньоквадратичної похибки. Запропоновано механізм вибору порядку ряду, глибини пам'яті та структури ядра залежно від типу сигналу (структурований, шумоподібний, комбінований), що дозволяє забезпечити компроміс між точністю моделювання та обчислювальною складністю.

За результатами чисельного моделювання для структурованих сигналів типу OFDM отримано зменшення ефективної ширини спектра на 1,9–8,6 % та відповідне зростання спектральної ефективності на 1,9–5 %. Показано, що використання базової моделі без адаптації для широкосмугових шумових сигналів призводить до розширення спектра та зниження ефективності, що підтверджує необхідність узгодження структури ядра з кореляційними властивостями сигналу. Визначено основні обмеження методу та обґрунтовано доцільність застосування попереднього декорелювання та зменшення спектральної надмірності.

Запропонований гібридний підхід забезпечує стабільну роботу в умовах змінної статистики сигналів, високого рівня завад і обмежених ресурсів обробки та може бути використаний у сучасних системах зв'язку, радіолокації та управління для підвищення ефективності використання спектра в режимі реального часу.

Ключові слова: ряди Вольтера; стохастичні сигнали; нелінійні системи; спектральна ефективність; ефекти пам'яті; адаптивна обробка сигналів; компенсація нелінійних спотворень; ефективна ширина спектра; енергетична концентрація; широкосмугові сигнали; зменшення спектральної надмірності.

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STATEMENT OF A SCIENTIFIC PROBLEM

In modern control systems, stochastic signals are gaining increasingly wide adoption due to their ability to ensure high reliability and interference robustness under complex jamming conditions.

Their statistical properties enable such systems to operate effectively under intentional interference and within a limited spectral resource. The application domain spans protected communication systems as well as advanced radar platforms employing technologies such as Spectrally Agile Frequency-Incrementing Reconfigurable (SAFIRE) radar, MIMO Noise Radar, and Advanced Pulse Compression Noise (APCN) radar.

However, unlike deterministic modulated signals, which typically exhibit a relatively uniform energy spectrum, stochastic signals possess a non-uniform spectral energy distribution.This uneven distribution increases the spectral redundancy of the stochastic signal, effectively broadening its occupied bandwidth and, consequently, reducing the system's spectral efficiency.

Classical methods for generating and processing stochastic signals–such as linear filtering or conventional spectral techniques–are unable to compensate for nonlinear distortions and memory effects inherent to the transmission channel. Empirical studies indicate that applying mathematical tools like Volterra series to describe nonlinear transformations in memory channels (e.g., for OFDM signals) can reduce spectral width and improve efficiency. Yet for wideband stochastic signals, these same methods often yield the opposite effect.

This inconsistency highlights the need for an improved method of stochastic signal generation–one capable of adapting the Volterra kernel parameters (order, memory depth, and structural form) to the statistical nature of the signal, and, when necessary, integrating with preprocessing techniques.

Therefore, developing an adaptive hybrid approach that provides stable improvements in spectral efficiency for both structured and unstructured stochastic signals is a highly relevant and timely task.

RESEARCH ANALYSIS

In studies dedicated to the modeling and processing of nonlinear systems, the effectiveness of applying Volterra series for modeling systems with memory is well established. At the same time, it is emphasized that increasing the nonlinearity order and memory depth leads to a corresponding growth in computational complexity [1, 2]. Therefore, simplified or structured alternatives are often used to address this problem, including block-oriented models such as Wiener, Hammerstein, and Wiener–Hammerstein systems [2], as well as parameter-reduction techniques–specifically, kernel decomposition into orthonormal Laguerre/Kautz bases or tensor factorizations such as PARAFAC and SVD [3, 4]. These methods reduce the number of parameters in nonlinear systems without a significant loss of accuracy; however, they frequently result in degraded performance under dynamic environmental conditions. Work [5] describes stochastic radar models incorporating multipath propagation, interference, "ghost" targets, and channel-memory effects.

Moreover, a number of studies emphasize the importance of regularization and the selection of an optimal model structure, which is particularly critical in tasks involving stochastic signals or memory-dependent models. Such approaches ensure stable kernel estimation and enable the model to operate effectively across a wide range of applications–from technical systems to financial models based on Volterra series [2, 3, 4, 13].

In the modeling of RF power-amplifier systems with memory effects, the Memory Polynomial (MP) and Generalized Memory Polynomial (GMP) models have gained widespread adoption, as they provide a practical balance between descriptive flexibility and the computational complexity of the identification procedure [6]. At the same time, classical Volterra series remain effective primarily in regimes of weak nonlinearity, whereas strong nonlinear distortion leads to convergence issues and practical difficulties in measuring high-order kernels. This has motivated the development of modified Volterra-based approaches ("modified Volterra") [4, 7, 8].

At present, limitations in the radio-frequency spectrum and the need for the simultaneous operation of radar systems and communication systems (IRC) have become a key factor driving stricter requirements for methods that reduce signal redundancy and ensure robust interference suppression in multipath environments [10, 11]. Current research efforts focus primarily on developing approaches capable of concentrating signal energy within the available bandwidth and improving detection performance even under high noise levels and intense interference conditions [7, 10].

Classical works by Rugh, Stenger, and Rabenstein provide the fundamental foundations of Volterra/Wiener modeling, including adaptive Volterra filters, Hammerstein/Wiener structures, and the key aspects of identifying nonlinear systems with memory. The development of this paradigm is further reflected in studies [3, 4, 12], which describe modern modifications of Volterra-based models (MP, GMP), issues of convergence and high-order kernel estimation, as well as practical applications in RF power amplifiers, communication channels, and linearity-enhancement tasks. These sources form the conceptual basis for analyzing nonlinear transformations in radio-engineering systems, where system memory, spectral distortion, and the efficiency of nonlinear compensation are critical factors.

In parallel, deep stochastic radar models reveal the complexity of real operating conditions: multipath propagation, interference, "ghost" targets, delays, and dispersion generate pronounced channel-memory effects for which static assumptions prove insufficient [5]. This underscores the need for adaptive approaches and careful selection of identification signals and criteria for systems with memory. At the same time, even simplified implementations of adaptive Volterra filters encounter difficulties related to the large number of coefficients, as well as the challenges of determining memory length and adaptation steps [2, 4]. Recent studies on time–frequency reconstruction further highlight the limitations of accounting for nonlinear components in spectral representations and the difficulty of controlling the complexity of high-dimensional kernels. Among promising research directions, authors emphasize tensor factorizations, regularization techniques, and adaptive procedures [14, 13].

Thus, existing approaches provide only partial solutions: Volterra series remain a universal modeling tool, block-oriented structures and MP/GMP models reduce complexity, and tensor factorizations optimize parametric dimensionality. However, none of these methods guarantees a stable improvement in spectral efficiency under varying statistical properties of signals and significant nonlinear distortions. This gap defines the relevance of the improved Volterra-based method proposed in this work [2, 4, 7, 13, 14, 18, 19,20].

THE PURPOSE OF THE WORK

The aim of this work is to improve the method of generating stochastic signals based on Volterra series by adapting the kernel parameters to the statistical nature of the input signals through adaptive kernel-coefficient updating according to the signal's statistical characteristics, and to experimentally evaluate the effectiveness of the proposed approach in terms of spectral efficiency compared with classical methods.

PRESENTATION OF THE MAIN MATERIAL AND SUBSTANTIATION OF THE OBTAINED RESEARCH RESULTS

In modern information and control systems, the generation and processing of stochastic signals take place under conditions of nonlinear distortions, memory effects, and limited spectral resources. Classical methods such as linear filtering or traditional spectral procedures do not provide the required level of adaptivity, as they fail to account for the complex interactions between signal samples, which are typical of channels with memory and structurally mixed signals.

As noted in [19,20], stochastic and chaotic signals possess wide spectra and highly non-uniform energy distributions, which complicates their processing using conventional linear methods and results in reduced spectral efficiency, even when such signals demonstrate high interference resistance. The authors emphasize that classical transforms–Fourier, wavelet, and Hilbert transforms–offer only partial adaptivity and cannot fully capture the intricate time–frequency relationships inherent to random processes. Therefore, they require replacement or augmentation by more flexible approaches. In contrast, Volterra series make it possible to describe signals not only through linear dependencies but also through their internal nonlinear interactions, which fundamentally shape the spectral structure of stochastic processes. Unlike classical linear transforms, the Volterra framework enables the modeling of memory effects, cross-correlations, and nonlinear dependencies between signal samples–properties characteristic of chaotic and stochastic signals. This capability provides a means for controlled spectrum shaping, suppression of excessive spectral components, and improved spectral efficiency in environments with nonlinear distortions.

At the same time, such systems must remain robust to changes in signal statistics and capable of adapting processing parameters to local or global variations in spectral content. As demonstrated in the study, the effectiveness of signal generation and subsequent processing is largely determined by the consistency of the preparatory stages–normalization, statistical stabilization, suppression of redundant components, and correction of distortions introduced both by the channel and by the generation stage. The coherence of these stages has a critical impact on the final spectral efficiency and signal reconstruction quality. Therefore, the practical implementation of the method requires not only an adequate model of nonlinear interactions but also a structured procedure for preprocessing, quality control, and adaptive refinement of parameters. The article emphasizes that such a staged, well-organized processes significantly enhances the efficiency of stochastic-signal processing and ensures optimal use of the spectral resource, even under unstable statistical conditions and interference-heavy environments.

In view of this, the work proposes an integrated framework for the generation and processing of stochastic signals that combines preliminary signal conditioning, nonlinear-interaction modeling, adaptive parameter updating, and spectral-efficiency evaluation. The structure of this framework is presented as a generalized algorithm that defines the sequence of stages for signal formation and processing, ensuring reproducibility and stable operation across a wide class of random processes.

From Figure 1, it can be seen that the implementation algorithm of the improved method for processing stochastic signals using Volterra series is both adaptive and iterative. The proposed algorithm is based on the classical workflow (normalization, Volterra model construction, parameter adaptation, and efficiency evaluation), but extends it with several important enhancements aligned with modern approaches to signal reconstruction and decorrelation.In particular, at the Volterra-model formation stage, the algorithm provides the option of applying tensor factorization of the kernels and regularization. This makes it possible to significantly reduce the number of parameters, stabilize the model in the presence of noise, and focus computations on the most significant spectral interactions.

The algorithm is further enhanced with a localized reconstruction mechanism that is activated only in regions of increased signal instability. The critical segments are identified using an instability indicator $K(t)$, and the reconstruction itself is localized through Gaussian windows. This approach, proposed in [17], avoids the excessive smoothing characteristic of global spectral methods and focuses the Volterra model specifically on the dynamic segments of the signal. In stable regions, global frequency-domain reconstruction is applied, which minimizes computational cost and ensures proper alignment of the kernel parameters with the physical and statistical properties of the signal. By combining local and global processing, the algorithm adaptively adjusts to changes in the signal structure and environmental conditions.

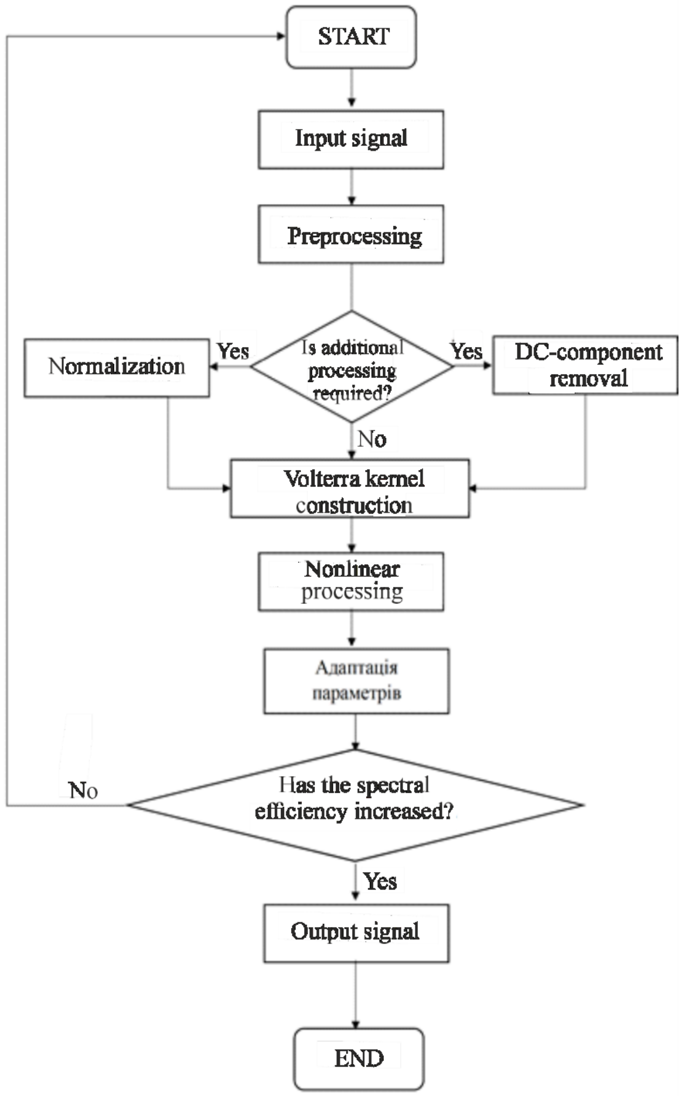

Fig.1. Block diagram of the algorithm for implementing the improved method of stochastic signal generation and nonlinear processing using Volterra series

The main stages of the algorithm are divided into blocks with logical checks and the possibility of backward transitions, which allows improving spectral efficiency even under challenging conditions. The algorithm begins with input-signal acquisition followed by a preprocessing block. Next, it is verified whether additional signal conditioning is required (e.g., normalization or DC-offset removal). If necessary, the algorithm switches to the corresponding processing blocks and then returns to the main workflow. After that, the Volterra model is constructed using parameters aligned with the characteristics of the signal. The processed signal then passes through the nonlinear-processing and parameter-adaptation block, which includes kernel refinement. A conditional evaluation of the result follows: if spectral efficiency has improved, the signal is forwarded to the output and the algorithm terminates. If no improvement is achieved, the algorithm performs a backward transition to the preprocessing stage, where the cycle is repeated with updated parameters.

This structure enables the algorithm to dynamically adapt to the signal type and environmental conditions, ensuring stable achievement of the target efficiency criteria without rigidly fixed configurations.

One of the promising tools employed is the Volterra series, which describe nonlinear distortions in the form (1):

$$y(t) = h_0 + n = \sum_{n=1}^{N} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} h_n(\tau_1, \dots, \tau_n) \prod_{k=1}^{n} x(t - \tau_k) d\tau_1 \dots d\tau_n \quad , \tag{1}$$

where h_0 , – the zero (constant) term of the series;

$h_n(\tau_1,\ldots,\tau_n)$, – the Volterra kernel (kernel function) of order n;

$x(t-\tau_k)$, – the input-signal value;

N – the maximum order of the series selected according to the model.

The order of the Volterra series determines the level of nonlinearity captured by the model. In practice, Volterra series of order N=2 or N=3 are most commonly used, as they allow effective modeling of the nonlinear effects typical of real-world channels and amplifiers.

The use of Volterra series for pre-compensation of distortions makes it possible to reduce spectral spreading and increase spectral efficiency.

Thus, the Volterra expansion can be viewed as a generalization of linear convolution for the case of weakly nonlinear systems with memory [16].

In modern wireless communication systems such as LTE and 5G, signals exhibit a complex hybrid structure. They combine deterministic components (e.g., synchronization signals, reference sequences, access-control blocks) with stochastic elements arising from the modulation of random traffic, interference, and multipath effects. This mixed nature of the signal introduces additional challenges in constructing a nonlinear processing model that must remain adaptive to the local characteristics of both the signal and the channel.

Such an approach is particularly relevant for modern information and control systems, where interference resilience and efficient spectrum utilization are critical performance requirements. In this context, Volterra-based signal formation and processing become especially valuable, as they enable adaptive modeling of both deterministic and stochastic components within dynamically changing environments. The main stages of the signal-generation and processing algorithm using Volterra series are as follows:

1. Preprocessing of the input signal.

The main objective of this stage is to structurally clean and align the input signal, as well as to configure the parameters of the model used for nonlinear processing. This stage includes the following components:

1. Signal normalization

The purpose of signal normalization is to bring the signal into a standard amplitude range. This procedure is aimed at eliminating the risk of distortion during subsequent processing, since stochastic signals generated in real environments typically exhibit arbitrary amplitude values. Normalization also ensures consistent processing across different types of signals.

Signal normalization is performed using two methods:

1. maximum-based normalization;
2. energy (root-mean-square) normalization.

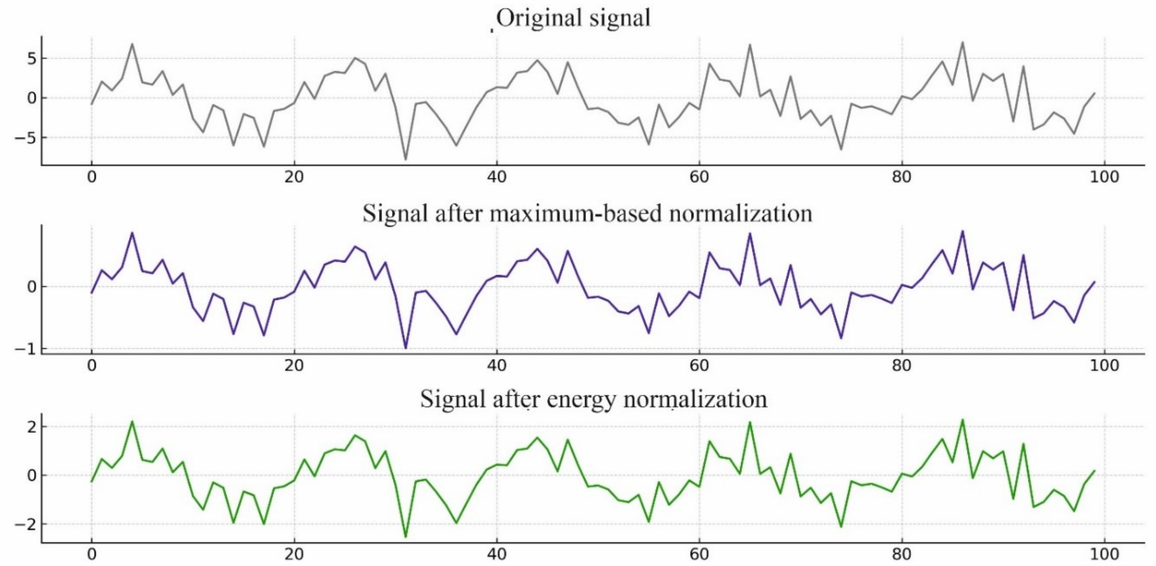

Fig.2. Comparison of stochastic-signal normalization methods

Figure 2 shows an example of a stochastic signal and the results of its normalization using the methods discussed above. The top plot presents a stochastic signal containing both a harmonic component and noise, which is typical for systems with unpredictable signal shapes.The middle plot illustrates the result of maximum-based normalization. In this method, all signal samples are divided by the maximum absolute value of the signal. As a result, the amplitude of the normalized signal is constrained to the $[-1,1]$, preventing numerical saturation and stabilizing subsequent nonlinear processing. The bottom plot shows the outcome of energy (RMS) normalization of the input signal. In this case, the signal is scaled so that its root-mean-square value (power) equals one. Energy normalization preserves the relative amplitude relationships while ensuring constant energy characteristics, which is beneficial for statistical signal analysis and evaluating system robustness to interference.

2. Removal of the DC component (centering).

Since in most practical cases signals do not have a zero mean, this results in the appearance of a zero-frequency component in the spectrum–the so-called DC component–which in nonlinear systems leads to parasitic harmonics, bias in power spectral density estimates, and distortions during nonlinear-compensation procedures.Centering the signal improves approximation accuracy, reduces spectral redundancy, and prevents artificial disturbances in the spectrum.The purpose of signal centering is to shift its values so that the mean of the signal becomes zero.

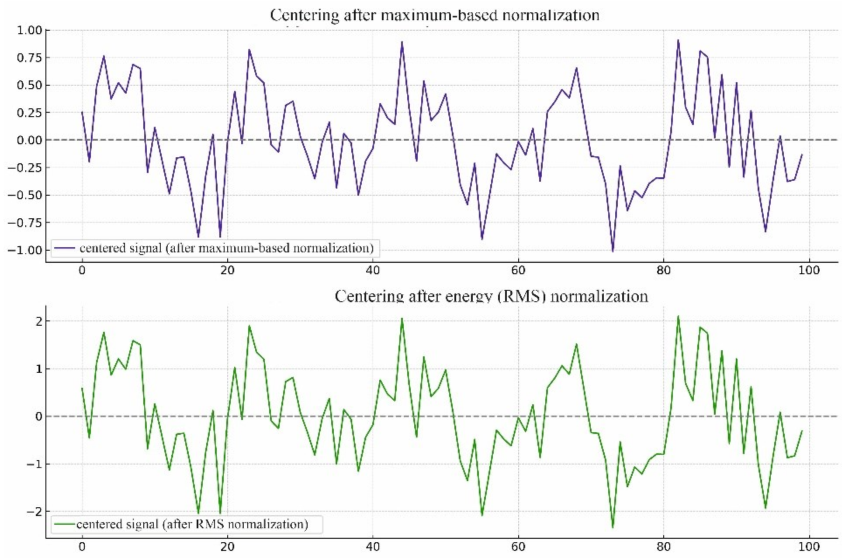

Fig.3. Signal centering after normalization

Figure 3 presents examples of the results of centering a normalized signal. The top plot shows the centered signal that was previously normalized using the maximum-based method. After removing the mean value, the signal oscillates symmetrically around the zero level. The bottom plot illustrates the signal after RMS (energy) normalization followed by centering. Despite having a higher amplitude range, its mean value is likewise reduced to zero, as indicated by the axis baseline.

Thus, it can be concluded that centering removes the DC component of the signal regardless of the normalization method applied..

2. Volterra model construction

At the Volterra-model construction stage, a mathematical model is built to describe the relationship between the input signal and its processed version. The key parameters of the model are the Volterra series order N, the memory depth M, and the kernel structure, all of which directly determine computational complexity, approximation accuracy, and the model's ability to capture the nonlinear characteristics of the channel.

As is well known, increasing the memory depth of an information system improves its ability to account for previous signal values, but at the same time increases computational complexity.

The components of this stage are:

1. Determination of the Volterra series order (N)

The most common case is the use of Volterra series with N=2, since further increasing the model order leads to a rapid growth in computational complexity with only a minor improvement in accuracy. This allows the mathematical model to be simplified to the form (2):

$$y(t) = h_0 + \int_{-\infty}^{+\infty} h_1(\tau)x(t-\tau)d\tau + \int\int_{-\infty}^{+\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 , \quad (2)$$

For the digital implementation of the output signal, the discrete form of the Volterra series is used, under the assumption that N=2 (3):

$$y[n] = h_0 + \sum_{i=0}^{M_1-1} h_1[i]\cdot x[n-i] + \sum_{i=0}^{M_2-1}\sum_{j=0}^{M_2-1} h_2[i,j]\cdot x[n-i]\cdot x[n-j] \quad (3)$$

where *y[n]* – the discrete-time realization of the signal;

$h_1[i]$, $h_2[i,j]$ – the kernel coefficients;
M_1 and *M_2* – the memory depths of the corresponding kernels.

Thus, using a second-order model makes it possible to effectively capture the most common nonlinear effects while maintaining controllable complexity, which is critically important for the practical implementation of the algorithm in real-time information systems.

In the developed algorithm, a second-order Volterra series (N=2), s also used, accounting for the following quadratic effects: amplitude compression, second-order harmonics, and mean-level shifts. This approach provides the required level of modeling accuracy for most physical nonlinearities encountered in communication channels or amplification elements, while avoiding excessive computational cost. Although higher-order Volterra series can, in principle, offer a more accurate representation of complex distortions, their practical implementation is associated with an exponential increase in the number of parameters. This, in turn, leads to a substantial rise in computational complexity.

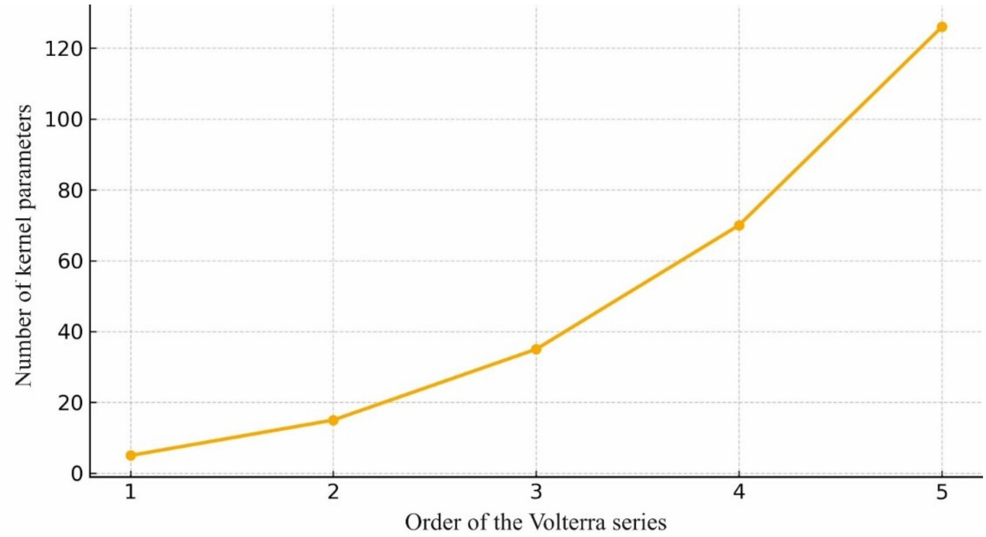

Fig.4 Dependence of the number of kernel parameters on the Volterra series order

Figure 4 shows that as the Volterra series order N increases, the number of kernel parameters grows nonlinearly. While a linear model (N = 1) requires only 5 coefficients, forтN = 2 this number increases to 15, and for N = 4 it exceeds 70. Such growth implies that the use of higher-order models significantly complicates computations and makes model adaptation more difficult, since estimating a large number of parameters requires substantially more statistical data. Therefore, employing a second-order Volterra series represents the best compromise between accuracy and complexity for real-time stochastic-signal processing tasks.

The effectiveness of using Volterra series with N=2 is supported by experimental results. The application of second-order Volterra series reduces the mean-square error (MSE) by an average of 43,6–57,8% compared with models based on first-order Volterra series, and decreases the mean-square deviation (MSD) to 20.55%. Additionally, an improvement of 10.3% in amplitude-response accuracy and 5.2% in phase-response accuracy has been observed.

<u>2. Selection of the memory depth M.</u>

The selection of the memory depth is performed during the model-initialization stage and directly affects the size of the input vector, the required memory, the number of kernel parameters, and the computational speed of the implementation. In Volterra series, the memory depth is determined by the spatial localization of the kernel energy and by the nature of the nonlinear interactions between the signal samples. For a second-order kernel *$h_2(k_1,k_2)$* the memory length is considered sufficient if the region of its significant coefficients covers 95–99% of the total kernel energy. The total energy of a second-order kernel is defined by (4):

$$E_N = \sum_{k_1=0}^{M-1} \sum_{k_2=0}^{M-1} \left| h_2(k_1,k_2) \right|^2 \tag{4}$$

where E_N – the total energy of the signal;;

$\left| h_2(k_1,k_2) \right|^2$ – the energy contribution of each kernel element;

M_1 and *M_2* – the memory depths of the kernel..

Accordingly, the sufficiency criterion for the selected memory depth is defined by condition (4):

$$\frac{\sum_{k_1=0}^{M-1}\sum_{k_2=0}^{M-1}\left|h_2(k_1,k_2)\right|^2}{\sum_{k_1=0}^{M_{\max}-1}\sum_{k_2=0}^{M_{\max}-1}\left|h_2(k_1,k_2)\right|^2}\geq\eta \text{ where } \eta\in\left[0.95;0.99\right] \quad (5)$$

where EN – the total energy of the signal;

$\left|h_2(k_1,k_2)\right|^2$ – the energy contribution of each kernel element;

M_1 and M_2 – the memory depths of the kernel..

η - energy sufficiency threshold.

Outside this region, the values of the Volterra kernel are small, and their contribution to the Volterra transform becomes negligible. Thus, the memory depth is effectively determined by the radius of the kernel's compact support in the index space *(k_1,k_2)*, which reflects how well the model can capture the nonlinear dependencies of the signal. In most cases, Volterra kernels synthesized for systems operating with stochastic signals and limited bandwidth exhibit a localized structure, with the majority of their energy concentrated within a region of relatively small dimensionality.

The optimal value of M is often determined empirically–based on preliminary analysis of the signal's correlation function or through adaptive tuning in real time. The parameter M defines the number of previous signal samples taken into account when forming the current output value. The larger the value of M, the broader the contextual window the model can incorporate–this may be advantageous, but may also introduce redundancy or overfitting, depending on the signal type. For noise-like or weakly correlated signals, a memory depth of M=1..2, s typically chosen, since such signals exhibit very weak dependencies between samples, and increasing the memory depth provides no additional information while simultaneously complicating the model and increasing the risk of spurious interactions between noise components.

Conversely, for periodic or structurally organized signals–particularly modulation compositions such as OFDM or QAM, which possess well-defined inter-sample dependencies–a memory depth of M=5…7 is recommended.

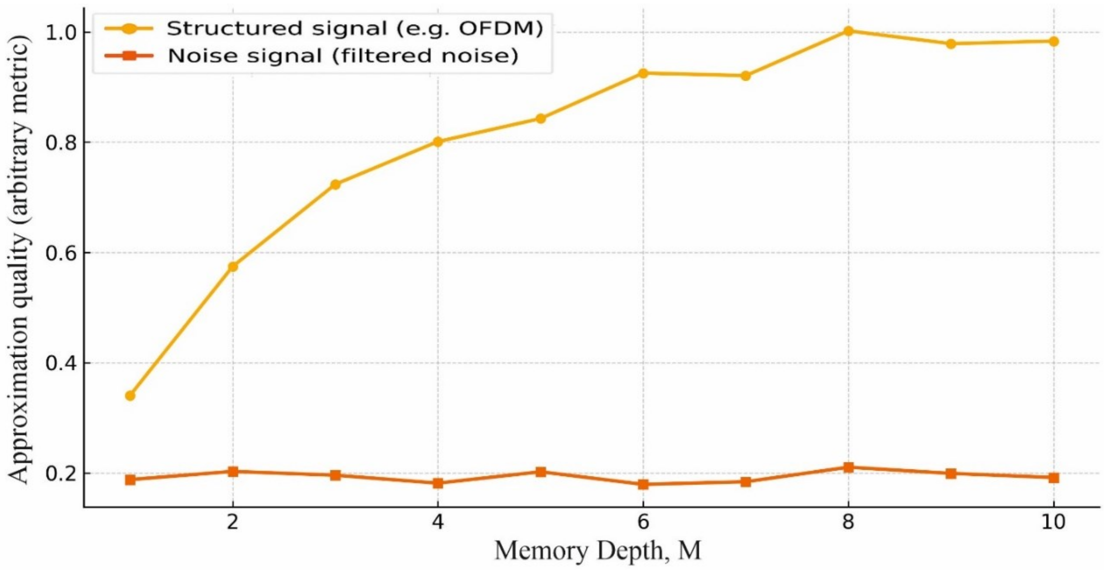

Fig.5. Dependence of approximation quality on memory depth *M*

Figure 5 shows that the approximation quality of the Volterra model depends on the selected memory depth M, and differs significantly for signals with different structures. For a structured signal (e.g., OFDM), a rapid increase in accuracy is observed as the memory depth grows, followed by saturation–further increasing M no longer provides noticeable improvement. This indicates the presence of a stable correlation structure that can be effectively captured by the model within a limited temporal window. In contrast, for a noise-like signal, the approximation quality remains almost unchanged with increasing M, which indicates the absence of persistent dependencies between samples. In such cases, a model with large memory depth only increases computational cost without improving performance.This confirms the practicality of using minimal memory depth M for processing random signals [15].

3. Selection of the Volterra kernel structure

As the baseline configuration, the algorithm employs a symmetric second-order Volterra kernel. This kernel structure accounts for quadratic interactions between signal samples in a manner similar to the full kernel, but eliminates duplicate terms arising from symmetry, thereby achieving high approximation accuracy without unnecessary computations. This type of kernel also reduces the number of independent coefficients compared with a full kernel–from 25 down to 15–and is well suited for typical signals used in communication systems. Moreover, it supports the use of adaptive adjustment mechanisms (e.g., MMSE-based adaptation or retention of only the most informative kernel elements). Therefore, this choice provides an optimal balance between accuracy and computational complexity while enabling further kernel adaptation according to the statistical properties of the signal..

3. Nonlinear signal processing:

Nonlinear signal processing based on the Volterra model makes it possible to address several practical challenges that arise under real transmission conditions. First, it provides compensation of nonlinear distortions caused by the properties of the channel or amplification elements, including amplitude compression, intermodulation effects, and phase distortions. Second, such processing helps reduce spectral redundancy generated by parasitic spectral components associated with quadratic and higher-order harmonics. Finally, due to the well-defined structure of the model, the kernel can be adaptively refined using current signal statistics, enabling flexible compensation even in dynamic or interference-rich environments.

For signals with a controlled structure, such as OFDM, numerical simulations were performed using the following parameters: 64 subcarriers, QAM-16 modulation, sampling rate f_s =960 kHz, and a third-order nonlinearity model with coefficient a_3 =0,2. After processing the signal using Volterra series, a reduction of spectral sidelobes by 8–10 dB was observed, which made it possible to narrow the effective bandwidth (i.e., the band containing 99% of the signal energy) from $B_{before} = 474{,}8$ kHz to $B_{after} = 465{,}9$ kHz (fig. 1.), according to $\Delta B \approx -2\%$ (5):

$$\frac{B_{після}}{B_{до}} \approx \frac{465{,}9}{474{,}8} \approx 0{,}98 \Rightarrow \Delta B \approx -2\%\,, \tag{5}$$

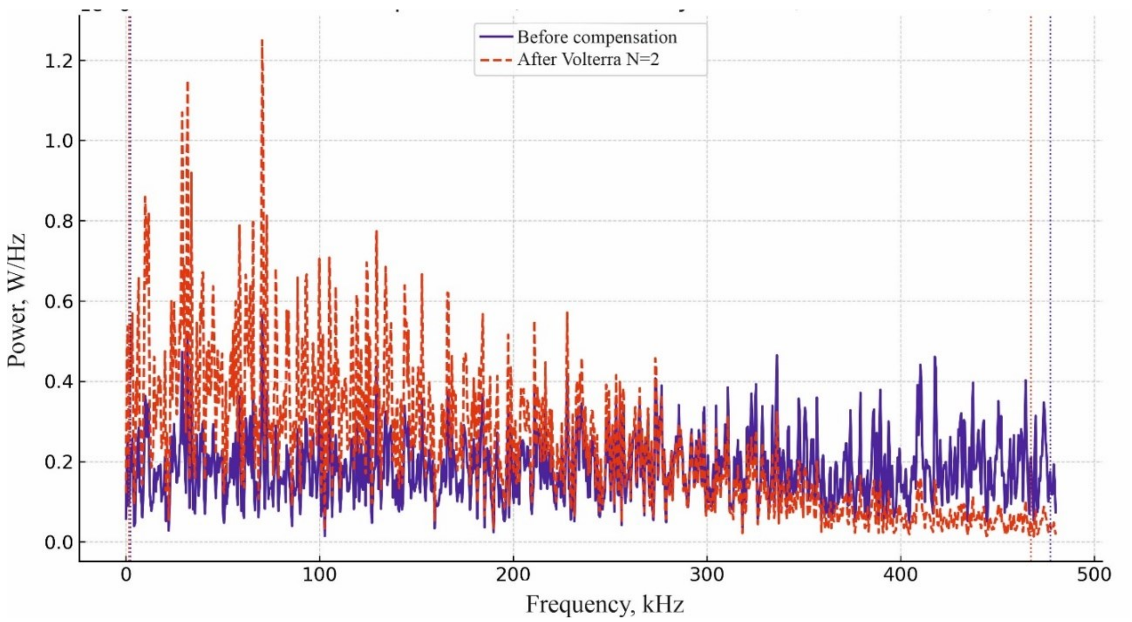

Fig.6. Spectrum of the OFDM signal before and after Volterra-series processing

Figure 6 illustrates the power spectral density of the OFDM signal (QAM-16, 64 subcarriers, f_s =960 kHz,), presented in a linear scale, which makes it possible to evaluate the actual amplitude values of power in W/Hz

The solid line corresponds to the signal after introducing third-order nonlinear distortions with coefficient α=0.2, while the dashed line represents the signal after compensation using the second-order Volterra model. The signal power is concentrated primarily within the range of $10^{-6}\ldots10^{-7}$ W/Hz, which is typical for digitally normalized signals.

The vertical dashed lines denote the boundaries of the effective bandwidth containing 99% of the signal energy. The reduction of this bandwidth after processing (from 474.8 kHz to 465.9 kHz) demonstrates a narrowing of the spectrum. Since the transmission rate *R* remained constant, the reduction in bandwidth resulted in a corresponding 1.9% increase in spectral efficiency, as given by (6):

$$\frac{\eta_{після}}{\eta_{до}} \approx \frac{R / B_{після}}{R / B_{до}} \approx \frac{B_{до}}{B_{після}} \approx 1{,}019 \tag{6}$$

$$\Delta\eta \approx (1{,}019 - 1) * 100\% \approx 1{,}9\%$$

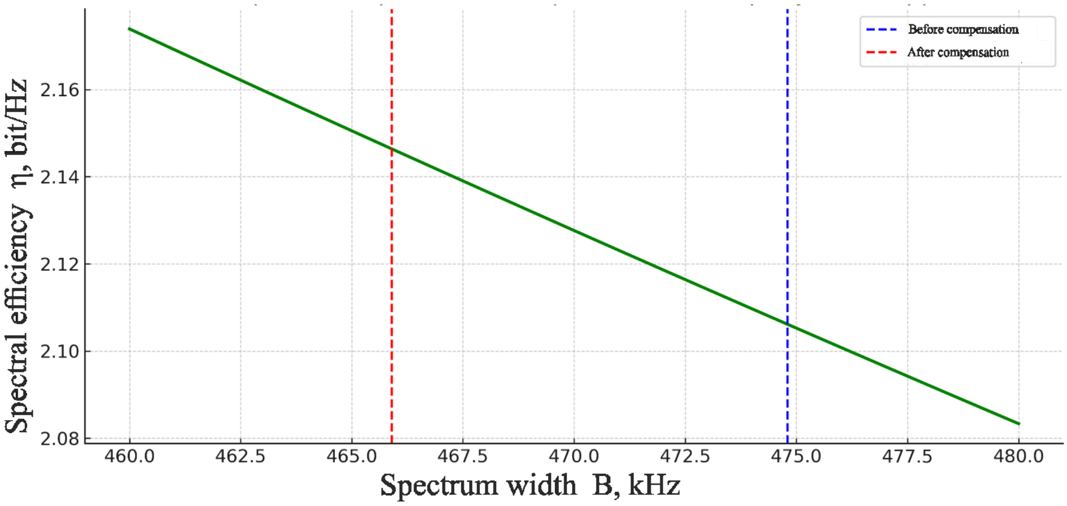

Fig.7. Change in the spectral efficiency of the OFDM signal before and after Volterra-series processing (N = 2)

As observed in Fig. 7, the reduction in spectral width achieved by second-order Volterra compensation results in a corresponding increase in spectral efficiency. Notably, even a minor bandwidth reduction (less than 10 kHz) yields an improvement of nearly 2%.

In contrast, for wideband stochastic signals (e.g., white noise or its realizations), which do not exhibit a pronounced harmonic structure, the simulation results were opposite. In the experiment with filtered white noise (fourth-order low-pass filter with normalized cutoff frequency 0.4), using 65 536 samples and a sampling rate of f_s =960 *kHz*, a third-order nonlinearity model with coefficient α =0,2 was applied.. The use of second-order Volterra series resulted in an increase in the signal's bandwidth from $B_{before} = 232\ kHz$ to $B_{after} = 266\ kHz$, corresponding to a 15% widening of the passband. Consequently, the spectral efficiency decreased by 13%.

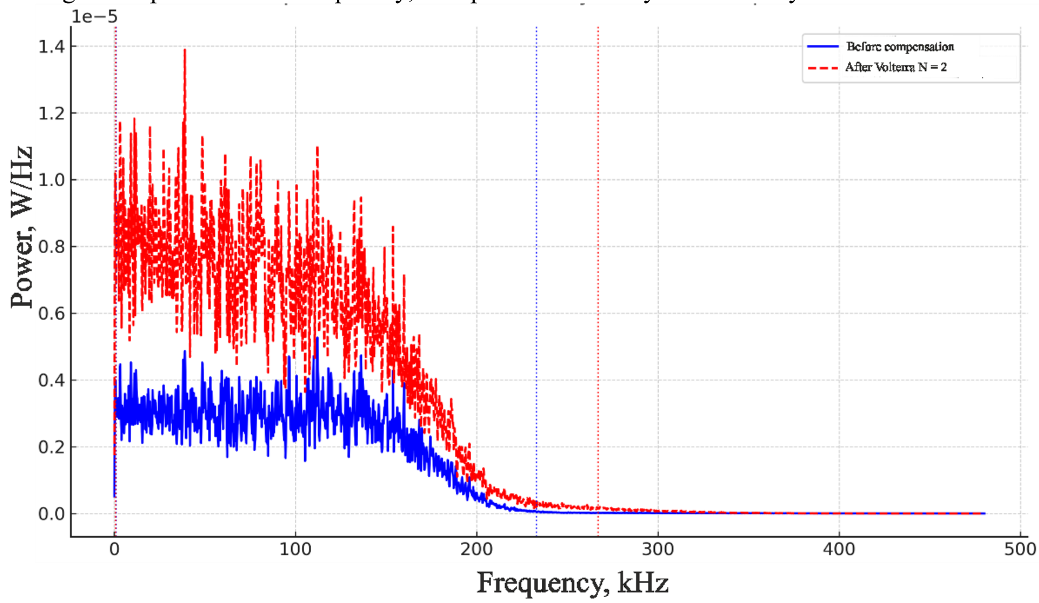

Fig.8. Spectrum of the stochastic signal before and after Volterra-series processing

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From Fig. 8, it can be observed that, after Volterra-series processing, the power of the stochastic signal increases in the high-frequency region and the overall spectrum becomes wider. This confirms that applying a basic nonlinear model without adaptation to the noise structure of the signal leads to an increase in spectral redundancy.

4. Parameter Adaptation.

In dynamic environments or when processing stochastic signals, a fixed Volterra kernel may not provide sufficient performance. In such cases, it is advisable to employ adaptive kernel tuning–a procedure that involves periodically updating the kernel coefficients based on the current characteristics of the signal.

The primary objective of adaptation is to minimize the error between the desired output and the model output, which is achieved using the Minimum Mean Square Error (MMSE) criterion (7):

$$MMSE = \min_{\{h_1, h_2\}} E\left[(y[n] - y[n])^2 \right], \quad (7)$$

where: *y[n]* – the actual (reference) or desired output;

y – the output generated by the Volterra model;

$h_1[i]$, $h_2[i, j]$– the first- and second-order kernel coefficients;

$E[\cdot]$ – the expectation operator (ensemble or sample mean).

At each adaptation step, a local error is also computed and used to update the kernel weights according to one of the optimization methods (e.g., gradient descent, LMS-type adaptive filters, RLS, etc.).

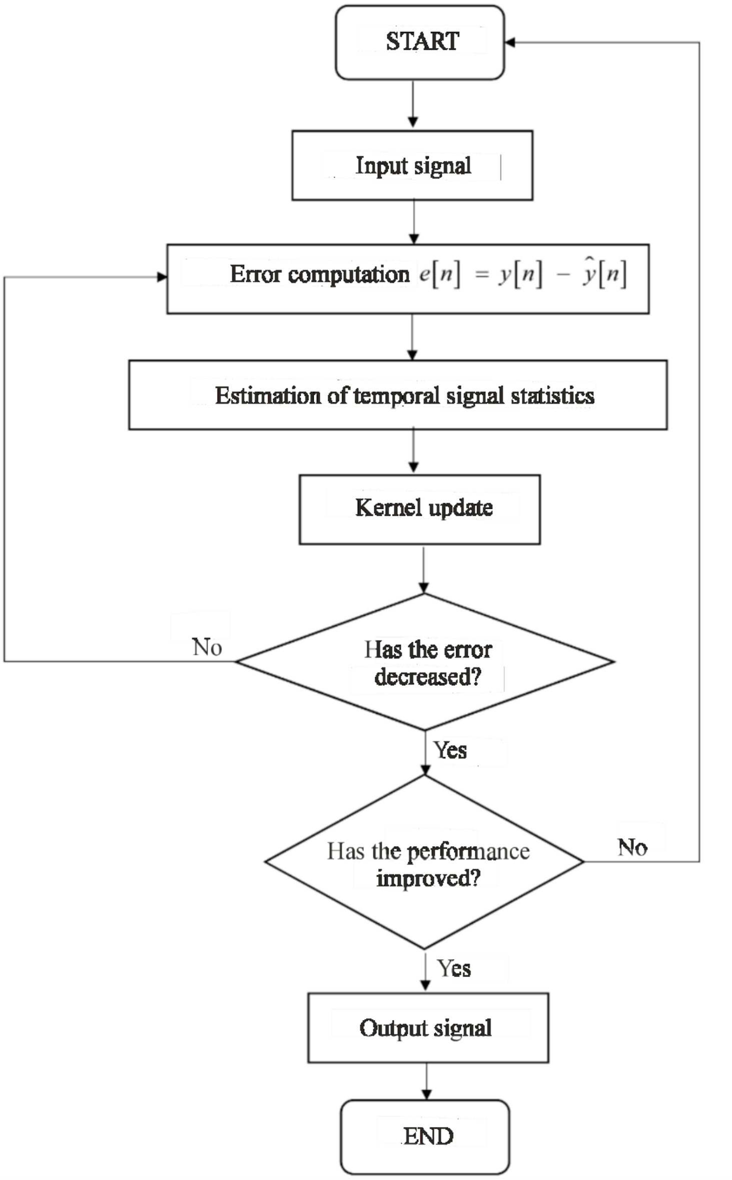

Fig.9. Block diagram of the Volterra kernel parameter adaptation

Figure 9 presents the block diagram of the Volterra kernel parameter adaptation process, implemented as a closed-loop cycle involving estimation, updating, and verification. The inputs to the algorithm are the current values of the input signal and the desired (reference) output. Based on the computed error, the statistical characteristics of the signal–such as covariance and cross-correlation–are estimated.

The kernel coefficients are then updated, for example, by solving a system of normal equations. The resulting model output is compared with the previous iteration, and if improvement is observed (i.e., reduced error or reduced spectral redundancy), the updated parameters are accepted. Otherwise, the cycle is repeated.

This approach enables dynamic adaptation to changes in the signal statistics or interference environment, ensuring stable real-time processing performance.

Adaptation is implemented as a cyclic process and includes the following stages:

1. Estimation of the current signal statistics:

- Computation of the covariance matrix: $C_x = E[x\, x^t];$
- Correlation between the output and the input: $r_{yx} = E[y[n] \cdot x]$.

2. Kernel-coefficient update: system of equations: $C_x \cdot h = r_{yx}$

3. Efficiency check: if the error decreases, the new parameters are retained; otherwise, the previous parameters are restored.

4. Decision stage: if the performance improves, the adaptation step is accepted; otherwise, the iteration is repeated.

The use of adaptation is particularly effective in systems where the signal structure may vary over time or under interference. This approach enables dynamic adjustment to environmental conditions while maintaining stable processing performance, without the need for restarting or manual reconfiguration.

5. Spectral-efficiency evaluation and correction

At the final stage of the adaptive signal-processing algorithm, the spectral efficiency is evaluated and, if necessary, the kernel parameters are corrected. This evaluation is performed by analyzing the power spectral density of the signal before and after processing, as well as by computing the effective spectrum width.

If the signal processing results in a reduction of B_{eff}, this indicates an improvement in spectral efficiency, since the main portion of the signal energy becomes concentrated within a narrower band. Conversely, if B_{eff}, increases, the kernel must be readapted by refining its coefficients, memory depth, or structural configuration. Thus, the effective spectrum width serves as a quality criterion for the processing stage and provides feedback for further refinement of the Volterra-model parameters.

Thus, the developed algorithm represents a hybrid system that combines a global frequency-domain model with tensor factorization and regularization in the stable regions of the signal, and a localized model with Gaussian windows in the critical intervals. This approach provides simultaneously high reconstruction accuracy, interference robustness, and acceptable computational efficiency, distinguishing it from traditional methods and confirming its scientific novelty [17].

The simulation results indicate that the effectiveness of stochastic-signal processing using the Volterra model depends on several factors:

1. the structure of the kernel (full, diagonal, symmetric, or sparse);
2. the memory depth M;
3. the series order N;
4. the statistical properties of the signal itself (presence of correlation, distribution type, energy spectrum);
5. the duration of the signal realization (sample length);
6. the level of nonlinearity of the system being approximated.

Table 1

Summary of the algorithm performance

№	Signal type	*N, M*	Signal length (samples)	*B* (before Volterra)	*B* (after Volterra)	*ΔB*, %	*Δη*, %
1	Filtered white noise (LPF=0.4)	3, 5	65536	180 kHz	207 kHz	+15	−13
2	Filtered white noise (example, RMS)	2, 3	1000	153 kHz	119 kHz	−22	+~18
3	OFDM (64-QAM, 960 kHz) (short realization)	2, 4	4096	162 kHz	148 kHz	−8.6	+~5
4	OFDM (64-QAM, 960 kHz) (full realization)	2, 4	>50000	474,8 kHz	465,9 kHz	−1,9	+1,9
5	Harmonic + noise	2, 3	2048	141 kHz	129 kHz	−8.5	+~6

As shown in Table 1, the application of the Volterra model does not always lead to an improvement in spectral efficiency. In the case of filtered white noise with a long realization (65 536 samples), using a third-order model with coefficient α=0,2, an increase in the effective bandwidth by 15% was observed, resulting in a ≈13% reduction in spectral efficiency (Figs. 6 and 8). This behavior is attributed to the absence of a pronounced correlation structure in the signal, as well as to the mismatch between the kernel shape of the model and the stochastic nature of the noise. In contrast, in simplified experiments with short segments of filtered stochastic signals (1000 samples) under milder parameter settings, a reduction of the spectrum by ≈22% was observed. This result should be interpreted as a local maximum caused by accidental correlation within a specific signal realization.

According to the obtained results, for signals of different types (harmonic, OFDM, noise-like), the reduction in spectrum width ranges from 1.9% to 9%, whereas for purely noise realizations an increase in spectrum width is observed.

Thus, the Volterra model does not provide an improvement in spectral efficiency when processing wideband stochastic signals, particularly in cases where their intrinsic statistical structure is not taken into account. Across several experiments, it was observed that when the kernel structure of the model is not aligned with the nature of the signal, not only may no improvement occur, but the spectral characteristics may even degrade–for example, due to spectrum widening or amplification of high-frequency components.

This allows formulating a set of essential limitations that must be taken into account when applying the model in practical scenarios:

1. the Volterra model is not universal for signals with unstructured or noise-dominated characteristics;
2. if the signal statistics do not correspond to the kernel structure, spectral overspreading may occur;
3. when processing short or high-noise realizations, a high variability of results is observed;
4. the model lacks a built-in mechanism for optimal kernel-structure selection based on the properties of the signal.

To mitigate these limitations, it is advisable to apply a preliminary signal transformation that enables:

1. reducing internal redundancy;
2. decorrelating the signal components;
3. isolating the components containing the main portion of the information.

CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

In this article, an improved method for generating stochastic signals for information and control systems based on a second-order Volterra model has been proposed. Numerical simulations demonstrated that the effectiveness of the method strongly depends on the consistency between the kernel structure and the statistical nature of the signal.

In particular:

1. or structured signals such as OFDM, depending on the realization length and model parameters, a reduction of the effective bandwidth in the range of 1.9% to 8.6% was observed, resulting in a corresponding increase in spectral efficiency of 1.9–5%;

2. for filtered white noise without preliminary transformation, the effective bandwidth increased by approximately 15%, which led to a ≈13% decrease in spectral efficiency.

These results allow formulating the main limitations of the method:

1. the Volterra model is not universal and is ineffective for signals with low correlation structure;
2. improper kernel selection may lead to spectral overspreading;
3. short or noise-dominated realizations exhibit high variability of results;
4. the method lacks a mechanism for automatic kernel–signal matching.

Furthermore, considering modern methodological trends, the algorithm can be enhanced by::

1. incorporating tensor factorization and regularization to reduce computational complexity and improve robustness to noise;

2. applying localized Gaussian windows and an instability indicator *K(t)* for more accurate reconstruction of signals with rapid transitions;

3. integrating machine-learning techniques for automatic adaptation of kernel parameters to the statistical properties of the signal.

Thus, the developed method is hybrid in nature and combines the classical foundation of the Volterra model with modern enhancements, ensuring practical effectiveness under diverse operating conditions of information and control systems.

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