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LEVKIN Dmytro
State Biotechnological University, Kharkiv
https://orcid.org/0000-0002-1980-4426
e-mail: dimalevkin23@gmail.com

APPLICATION OF CALCULATION METHODS FOR SOLVING APPLIED PROBLEMS OF HEAT AND MASS EXCHANGE IN COMPLEX SYSTEMS

The accuracy of the calculation and optimization of the objective function and its parameters when solving applied engineering problems depends on the accuracy of the formulation of the main optimization problem, calculation and applied optimization problems, as well as the accuracy of computational methods for their implementation. An increase in the considered features of applied optimization problems will complicate the setting and methods of implementing boundary value problems. Thus, for the implementation of modernized boundary value problems, it will be necessary to apply several computational methods that will create a computational structure. The main condition for constructing a physically based boundary value problem is to find and justify the conditions for the existence of a unique solution. To increase the efficiency of the use of methods of calculation and optimization of technical parameters, it is necessary to increase the number of considered features of calculation and applied optimization mathematical models for heat and mass transfer in technical systems. Along with the construction of boundary value problems, it is important to define and justify the conditions for the existence of a single solution.

The research article deals with some aspects of solving applied problems of heat and mass transfer in technical systems. Nonlocal boundary value problems for inhomogeneous and homogeneous pseudodifferential equations in partial derivatives with integral boundary conditions are considered, methods of solving a nonlocal inhomogeneous boundary value problem are proposed, and the correctness conditions of this problem in the class of infinitely differentiable generalized functions of power growth are defined and proven. Proved conditions for the existence of a correct problem for pseudo-differential equations with an integral boundary condition. The research of this article should be applied for controlling possible risks when solving applied problems in technical systems, biotechnology and veterinary medicine.

***Keywords**: applied engineering problems, calculation methods, heat and mass transfer, boundary value problem.*

ЛЕВКІН Дмитро
Державний біотехнологічний університет

МЕТОДИ РЕАЛІЗАЦІЇ ПРИКЛАДНИХ ЗАДАЧ ТЕПЛОМАСООБМІНУ ДЛЯ СКЛАДНИХ СИСТЕМ

Точність розрахунку і оптимізації функції мети та її параметрів при розв'язанні прикладних інженерних задач залежить від точності постановки основної оптимізаціної задачі, розрахункових і прикладних оптимізаційних задач, а також точності обчислювальних методів для їх реалізації. Збільшенння врахованих особливостей прикладних задач оптимізації ускладнить постановку і методи реалізації крайових задач. Так, для реалізації модернізованих крайових задач доведеться застосувати декілька обчислювальних методів, які створять обчислювальну структуру. Головна умова для побудови фізично-обгрунтованої крайової задачі – це знайти та дослідити умови існування єдиного розв'язку. Для збільшення ефективності використання методів розрахунку і оптимізації технічних параметрів потрібно збільшити кількість врахованих особливостей розрахункових і прикладних оптимізаційних математичних моделей для процесів тепломасообміну в технічних системах. Поряд з побудовою крайових задач для цього важливим є визначення і обгрунтування умов існування єдиного розв'язку.

В статті досліджені деякі аспекти розв'язання прикладних задач тепломасooобміну в технічних системах. Розглянуті нелокальні крайові задачі для неоднорідних і однорідних псевдодиференціальних рівнянь в частинних похідних з інтегральними крайовими умовами, запропоновані методи розв'язання нелокальної неоднорідної крайової задачі, а також визначені і доведені умови коректності цієї задачі в класі нескінченно-диференційованих узагальнених функцій степеневого зростання. Доведені умови існування коректної задачі для псевдодиференціальних рівнянь з інтегральною крайовою умовою. Дослідження цієї статті доцільно застосувати для контролінгу можливих ризиків при розв'язанні прикладних задач в технічних системах, біотехнології та ветеринарії.

Ключові слова: прикладні інженерні задачі, методи розрахунку, тепломасообмін, крайова задача.

Formulation of the problem

The study of the theoretical foundations of the construction and implementation of applied engineering problems of heat and mass transfer is connected with the difficulties of their formalization. To increase the accuracy of the solution of boundary value problems, it is necessary to increase the number of factors of simulated processes, which will complicate the existing mathematical models with differential equations. Summarizing the research of many scientists, it can be stated that non-local boundary value problems with systems of pseudo-differential equations are used for mathematical modeling and analysis of the state of many technical systems that contain sources of thermal action. However, the correctness of these boundary value problems cannot be guaranteed.

In the article, homogeneous and inhomogeneous boundary-value problems for pseudo-differential equations with integral conditions on the segment of the real axis are considered, their specific properties are analyzed, based on which, the solution of the boundary-value problem for a system of inhomogeneous pseudo-differential equations with an integral boundary condition is obtained. The correctness conditions of the inhomogeneous boundary value problem in the space of infinitely differentiable generalized functions are presented and substantiated in detail. It is advisable to apply the research of this article to prove the correctness conditions of other multifactorial mathematical models when solving applied problems of heat and mass transfer. This will lead to an increase in the accuracy of the main optimization criterion, a clearer separation of the optimized area and delineation of its boundaries, which will allow a more precise analysis of options with the aim of choosing the optimal.

Analysis of the latest research

In publications [1, 2], the peculiarities of using information and control technologies and control systems to reduce useless heat losses in buildings and construction structures are investigated. To increase the energy efficiency of buildings, measurements of the thermal resistance of external enclosing structures were carried out [3]. Using the correctness conditions of boundary value problems with systems of differential equations in partial derivatives, full controllability conditions for systems with distributed parameters were obtained in publications [4, 5]. Conditions for increasing the efficiency of cloud infrastructure process optimization are proposed in publications [6, 7]. To ensure uniform distribution of thermal effects on machine parts, the authors of the publication [8] proposed mathematical models and methods of rational placement of objects in a closed 3D area. The publication [9] proposed mathematical models and methods for fast image retrieval in the information storage repository.

The purpose of the work is: to increase the accuracy of the implementation of calculation mathematical models for heat and mass exchange processes in technical systems.

Presenting main material

Calculation mathematical model for technical systems that contain sources of thermal load:

$$\frac{\partial u(x,t)}{\partial t} = A\left(t, \frac{\partial}{\partial x}\right) u(x,t) + f(x,t)\,, \tag{1}$$

which satisfies the integral boundary conditions on the segment of the real axis:

$$\int_0^T B\left(t, \frac{\partial}{\partial x}\right) u(x,t)\, d\mu(t) = 0\,. \tag{2}$$

Fourier's transformation of the boundary value problem (1)–(2):

$$\frac{\partial \tilde{u}(s,t)}{\partial t} = A(t,s)\tilde{u}(s,t) + \tilde{f}(t,s)\,, \tag{3}$$

$$\int_0^T B(t,s)\tilde{u}(s,t)\, d\mu(t) = 0\,. \tag{4}$$

To substantiate the correctness conditions of the boundary value problem (1)–(2), consider a homogeneous boundary value problem:

$$\frac{\partial u(x,t)}{\partial t} = A\left(t, \frac{\partial}{\partial x}\right) u(x,t)\,. \tag{5}$$

Integral boundary conditions:

$$\int_0^T B\left(t, \frac{\partial}{\partial x}\right) u(x,t)\, d\mu(t) = \varphi(x,t)\,. \tag{6}$$

By substituting the solution of equation (5) $\tilde{u}(s,t) = \left(\exp\int_0^t A(\tau,s)\, d\tau\right) \cdot \psi(s)$ into the boundary conditions (6), we obtained:

$$\int_0^T B(t,s)\left(\exp\int_0^t A(\tau,s)\, d\tau\right) d\mu(t) \cdot \psi(s) = \tilde{\varphi}(s)\,. \tag{7}$$

Equation (7) is true under the condition if $\int_0^T B(t,s)\left(\exp\int_0^t A(\tau,s)\, d\tau\right) d\mu(t) \neq 0$.

For further research, we will apply the Greene's function:

Continuously differentiable on $[0,\tau)\cup(\tau,T]$ function $G(s,t,\tau)$ is called a Greene's function if:

$G(s,\tau+0,\tau)-G(s,\tau-0,\tau)=1$ on $t\in[0,\tau)\cup(\tau,T]$, $\frac{\partial}{\partial t}G(s,t,\tau)=G(s,t,\tau)$ for $t\in[0,\tau)\cup(\tau,T]$.

$\int_0^T B(s,t)G(s,t,\tau)d\mu(t)=0$, for any point of the segment $[0,T]$. Note that the existence of the Greene's function determines the existence of a single solution of the inhomogeneous boundary value problem (3)–(4).

Using the results of publications [10, 11], we found that the correctness of the homogeneous boundary value problem (5)–(6) determined the correctness of the nonhomogeneous boundary value problem (1) with the integral condition (2) in the space of infinitely differentiable generalized functions of exponential growth.

It is advisable to apply the research results, for example, to justify the conditions of the correctness of the boundary value problem with the equation of thermal conductivity, which records the state of a spherical material under the action of thermal load sources:

$$\rho c\frac{\partial T}{\partial t}-\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right)+q(r,t)=0 \tag{8}$$

with integral boundary conditions (2) and initial conditionson:

$$T(r_0,t_0)+T(r_n,t_n)=T_{sym}, \tag{9}$$

where ρ, c, λ – coefficients responsible for the thermophysical properties of the modeled system;

$T=T(r,t)$ – harmful temperature;

$q(r,t)$ – density function of the temperature distribution.

Another example of a correct boundary value problem is the boundary value problem for the system of differential equations of thermal conductivity used to calculate the temperature of the laser effect on the embryo:

$$\begin{cases}\rho_1 c_1\frac{\partial T_1}{\partial t}-\lambda_1\left(\frac{\partial^2 T_1}{\partial r^2}+\frac{2}{r_1}\frac{\partial T_1}{\partial r}\right)+q_1=0;\\ \rho_2 c_2\frac{\partial T_2}{\partial t}-\lambda_2\left(\frac{\partial^2 T_2}{\partial r^2}+\frac{2}{r_2}\frac{\partial T_2}{\partial r}\right)+q_2=0;\\ \rho_3 c_3\frac{\partial T_3}{\partial t}-\lambda_3\left(\frac{\partial^2 T_3}{\partial r^2}+\frac{2}{r_3}\frac{\partial T_1}{\partial r}\right)+q_3=0,\end{cases} \tag{10}$$

which satisfy the boundary conditions (2), (9).

In order to improve the quality of controlling the thermal distribution in the boundary value problem, the equalities of the thermal contact of the layers in the embryo were used:

$$\begin{cases}T_1(r_1,t_1)=T_2(r_2,t_2), \quad -\lambda_1\frac{\partial T_1}{\partial r}=-\lambda_2\frac{\partial T_2}{\partial r};\\ T_2(r_2,t_2)=T_3(r_3,t_3), \quad -\lambda_2\frac{\partial T_2}{\partial r}=-\lambda_3\frac{\partial T_3}{\partial r};\\ T_3(r_3,t_3)=T_4(r_4,t_4), \quad -\lambda_3\frac{\partial T_3}{\partial r}=-\lambda_4\frac{\partial T_4}{\partial r}.\end{cases} \tag{11}$$

Using the results of the publication [10], we found that the solutions of the boundary value problems for the differential equations (8), (10) satisfy the correctness conditions and therefore these boundary value problems are correct in the space of infinitely differentiable generalized functions of power growth.

Conclusions

Effective design and analysis of the functioning of technical systems requires the development of new applied optimization mathematical models and numerical methods of their implementation, or the improvement of existing ones by increasing the accuracy of the implementation of boundary value problems and the search process for the optimization of technical parameters. This will complicate the methods of formalizing mathematical models for applied problems. Due to the fact that it is quite often necessary to apply several computational methods for the implementation of modernized mathematical models, the first priority is the task of building a physically based mathematical model that has a single solution.

The article defines and substantiates the correctness conditions of nonlocal boundary value problems for a system of inhomogeneous pseudo-differential equations with an integral boundary condition in the space of infinitely differentiable, generalized functions. The results of research should be applied to improve existing modeling devices and hardware and software for monitoring possible risks during the operation of technical systems. This will increase the accuracy of the loss temperature calculation when implementing applied heat and mass transfer problems.

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